Competition in two-layer multiplex networks

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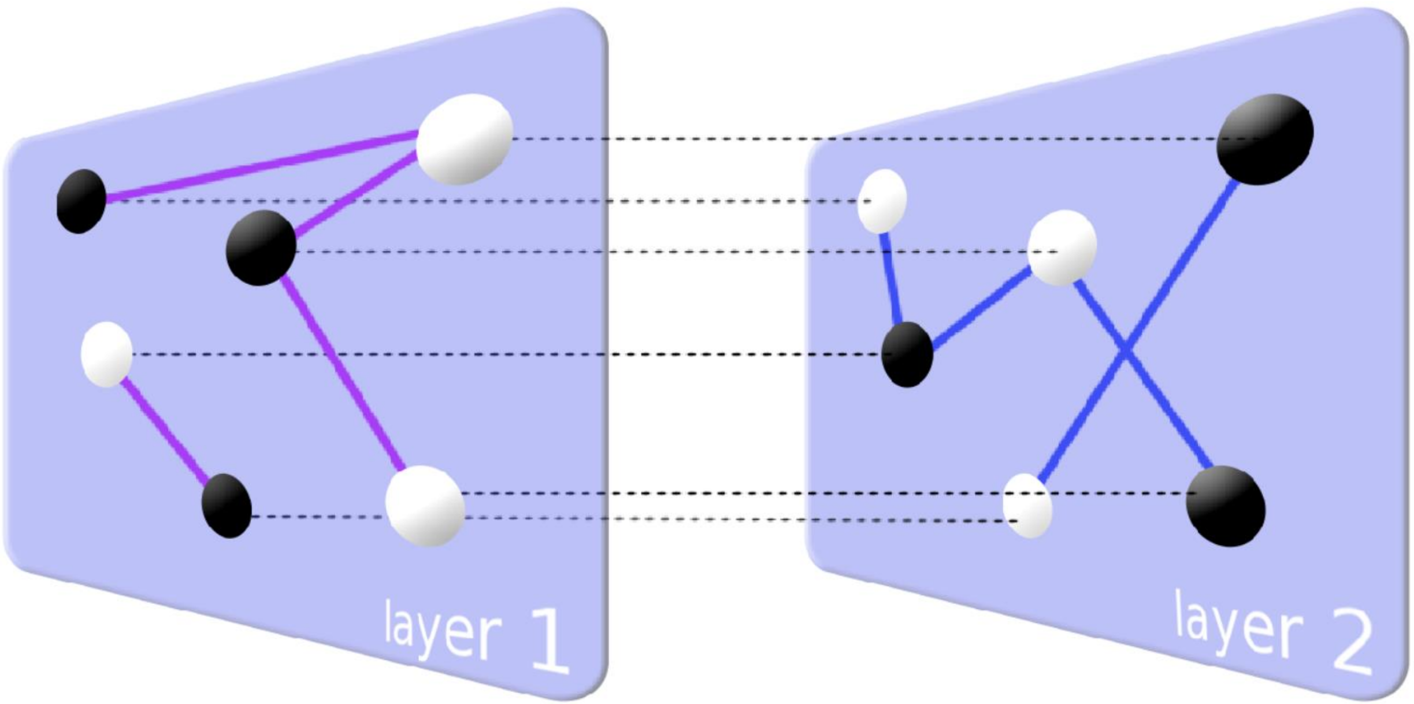
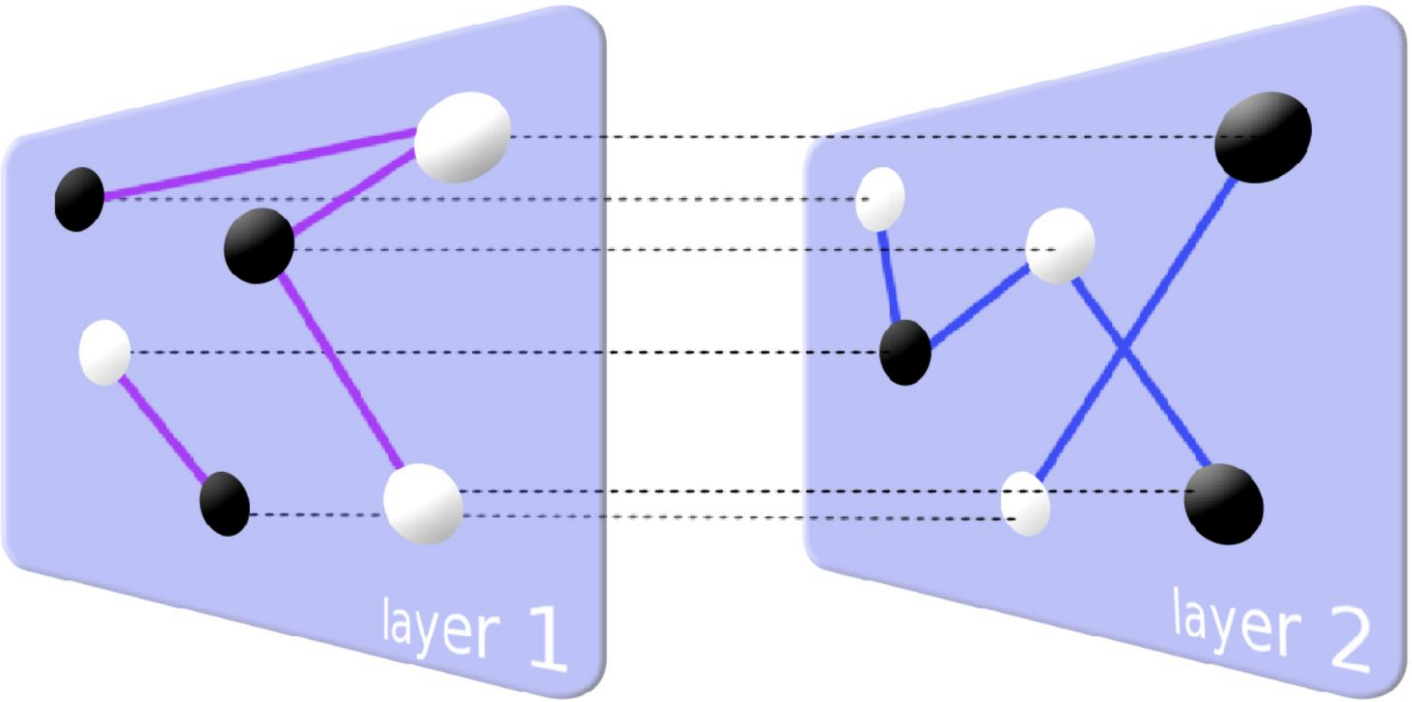
CCS / DOOCN-XI 2018 Thessaloniki

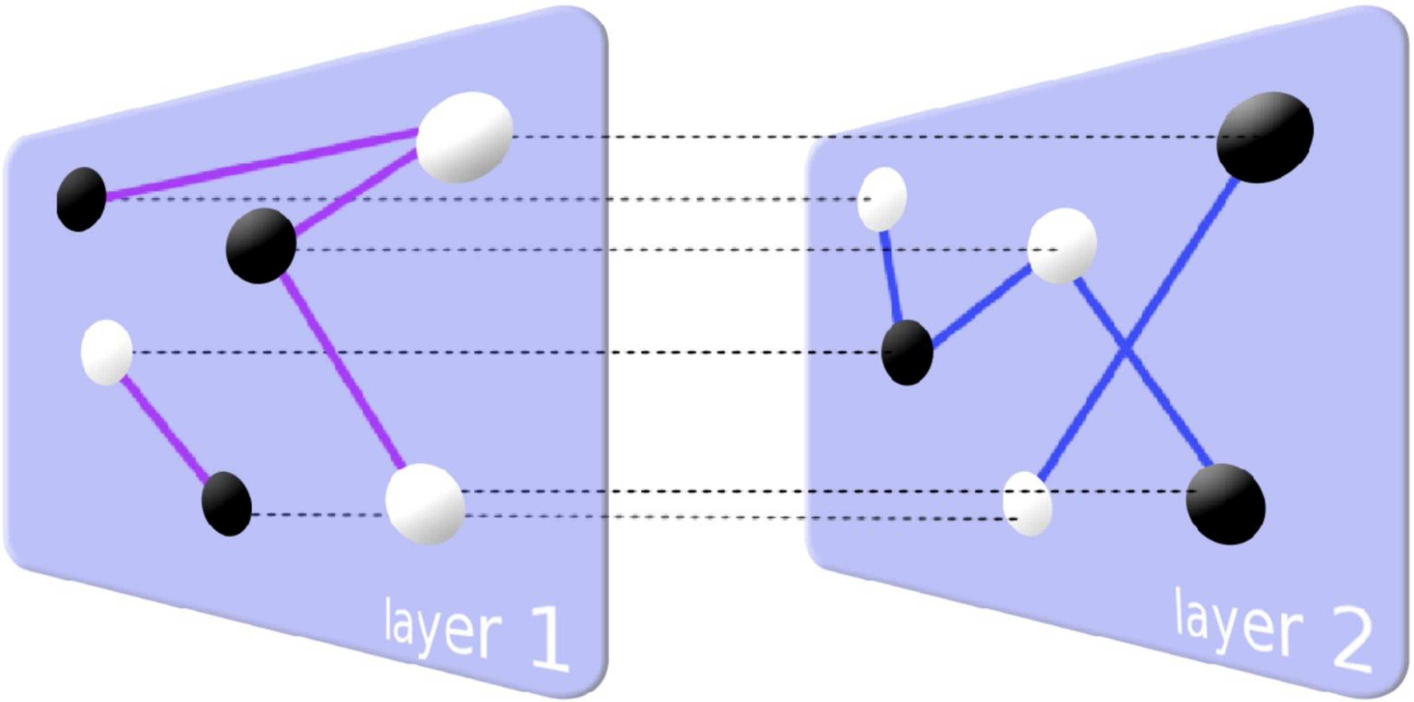
Outline

- Motivation
- Competition dynamics
- Ground State
- Optimization
- Results

- Structure
  - Multiplex network




layer 1
layer 2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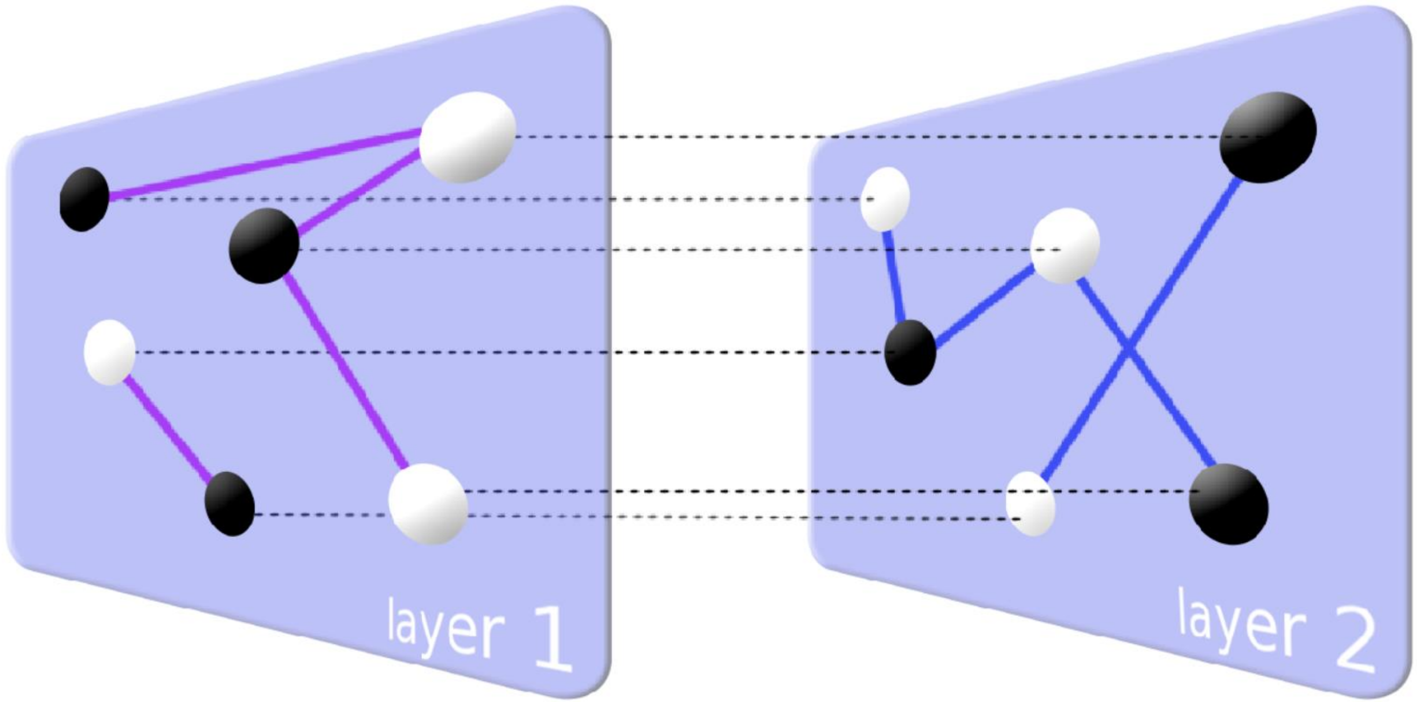

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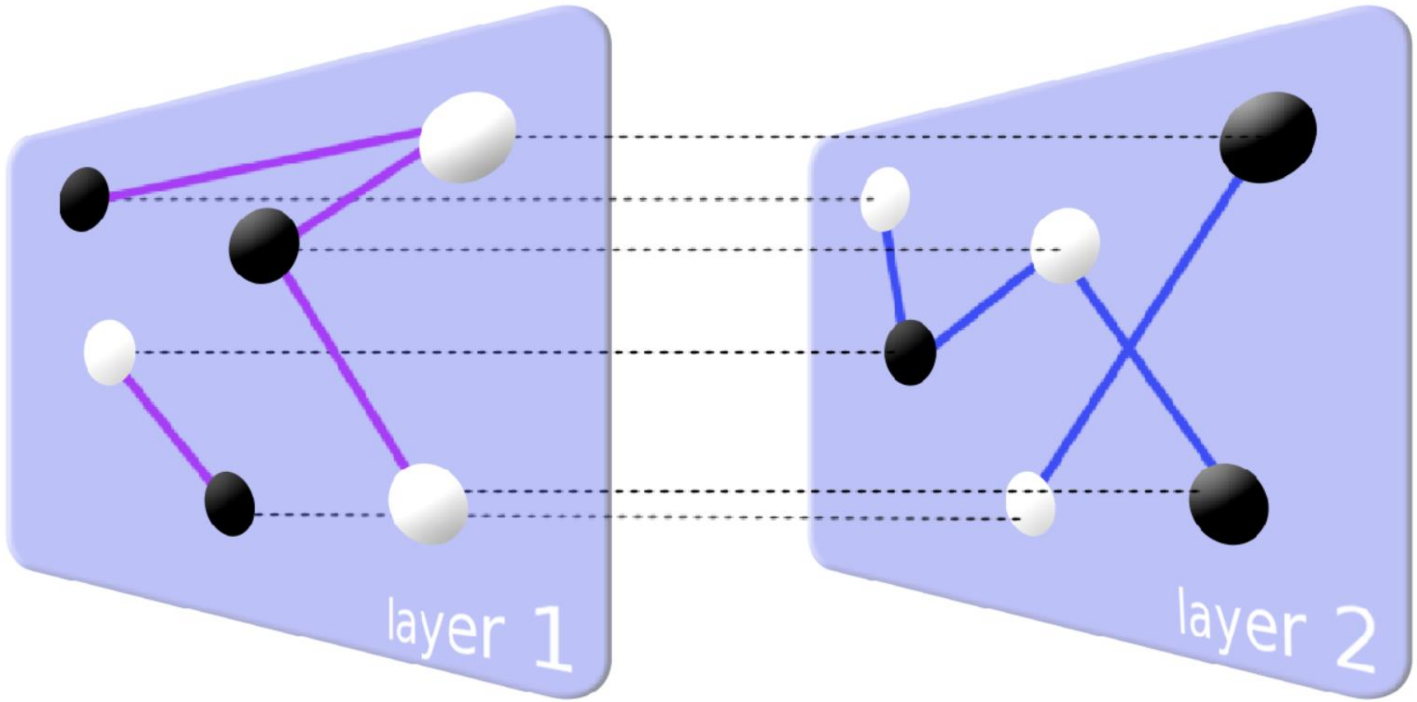
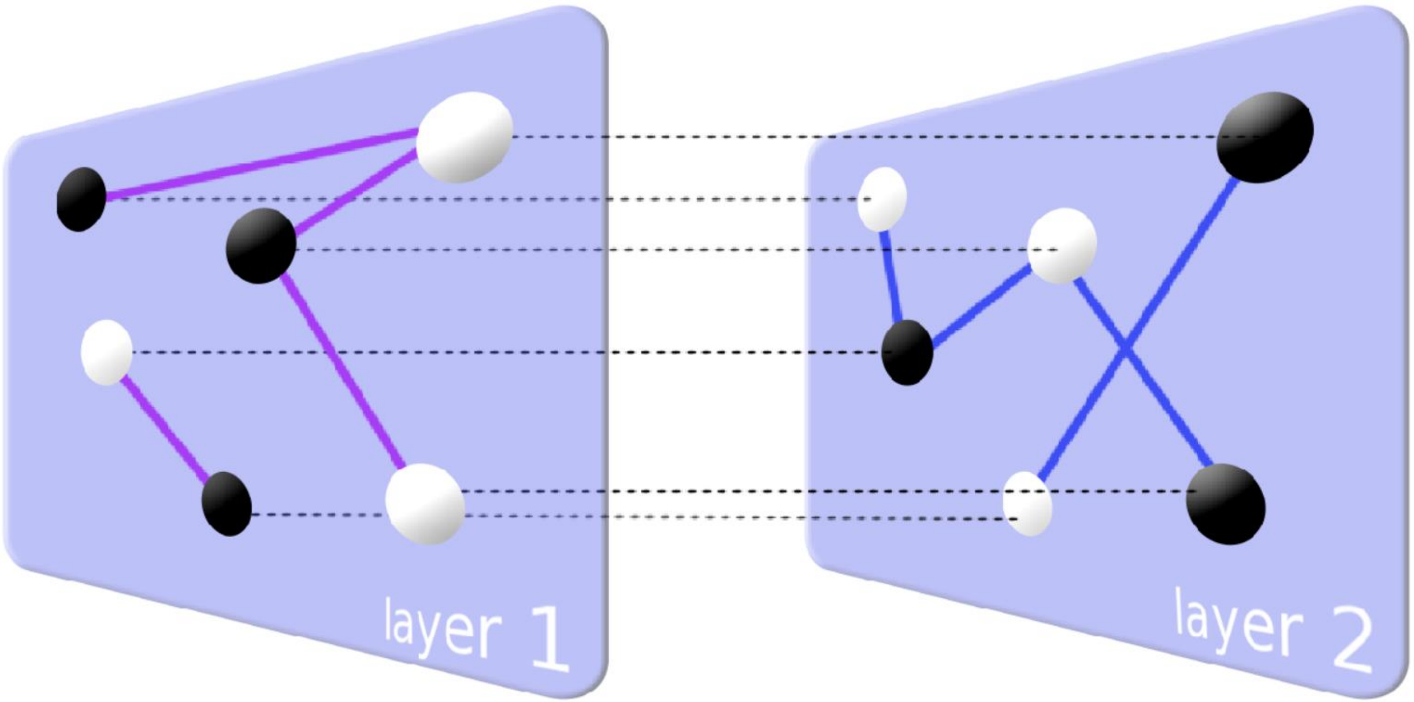


ZARA

GIORGIO ARMANI

Competition in two-layer multiplex networks



- Dynamics
  - Individuals can choose between several alternatives

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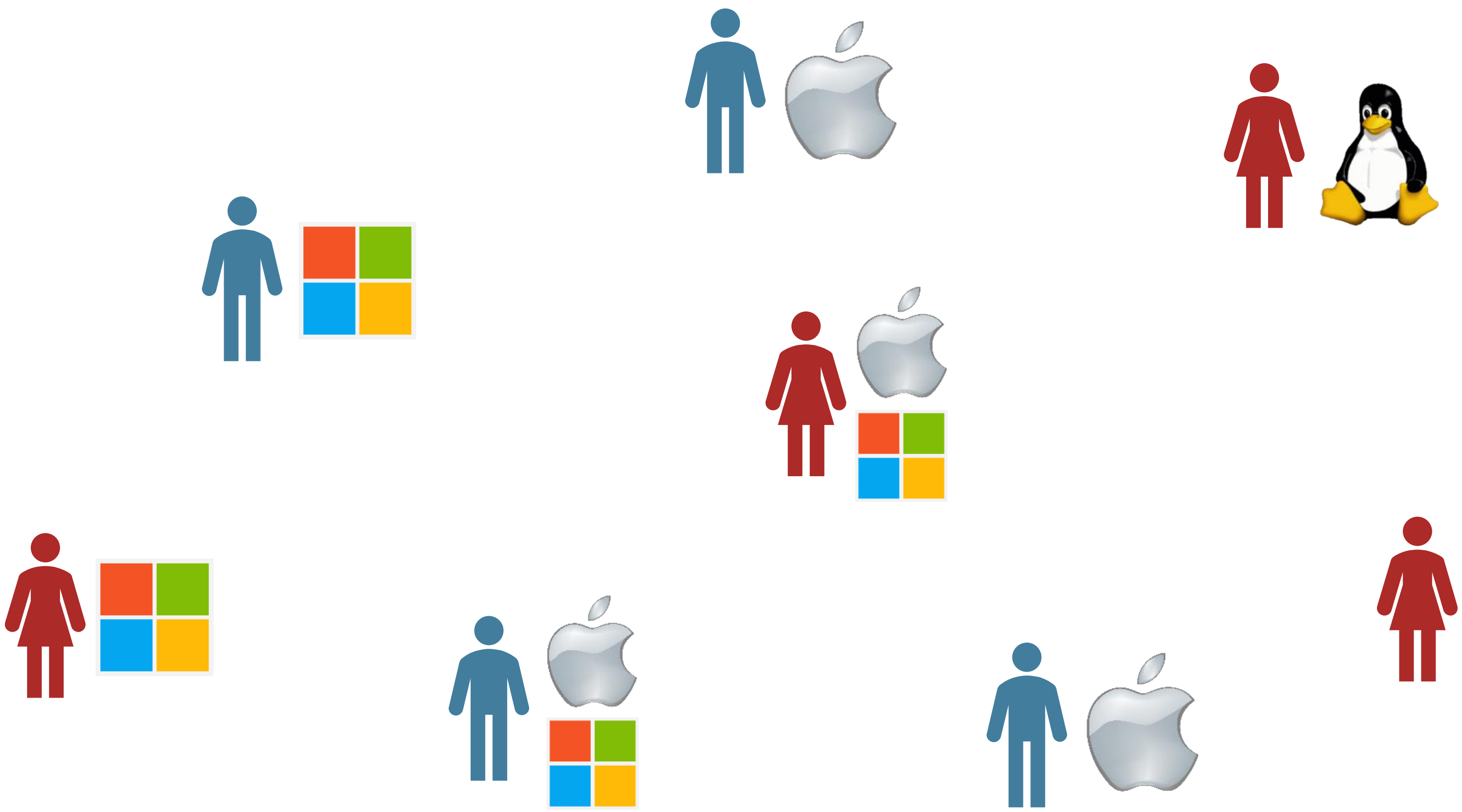

- Dynamics
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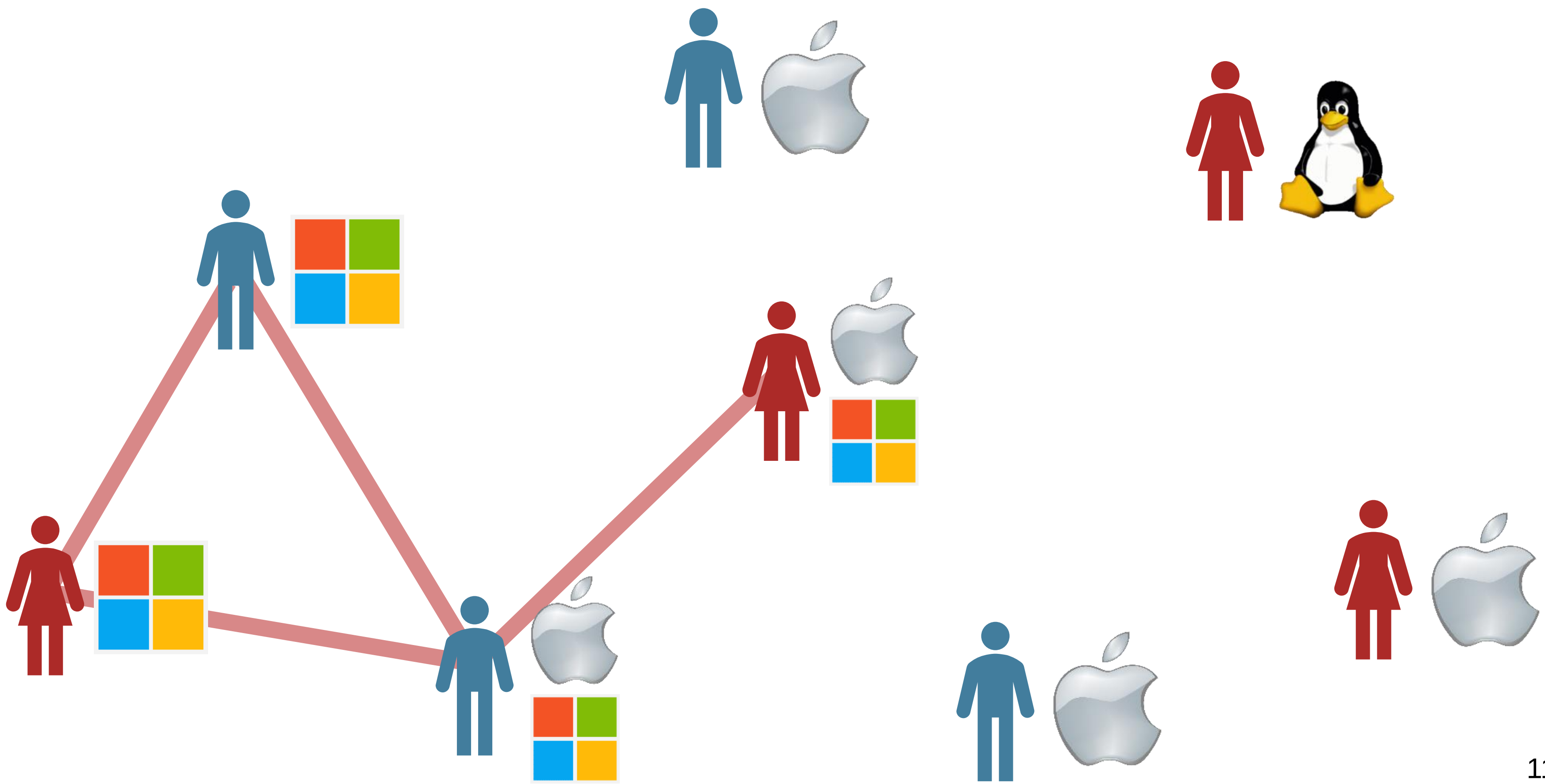
  - Sharing an alternative with peers is beneficial



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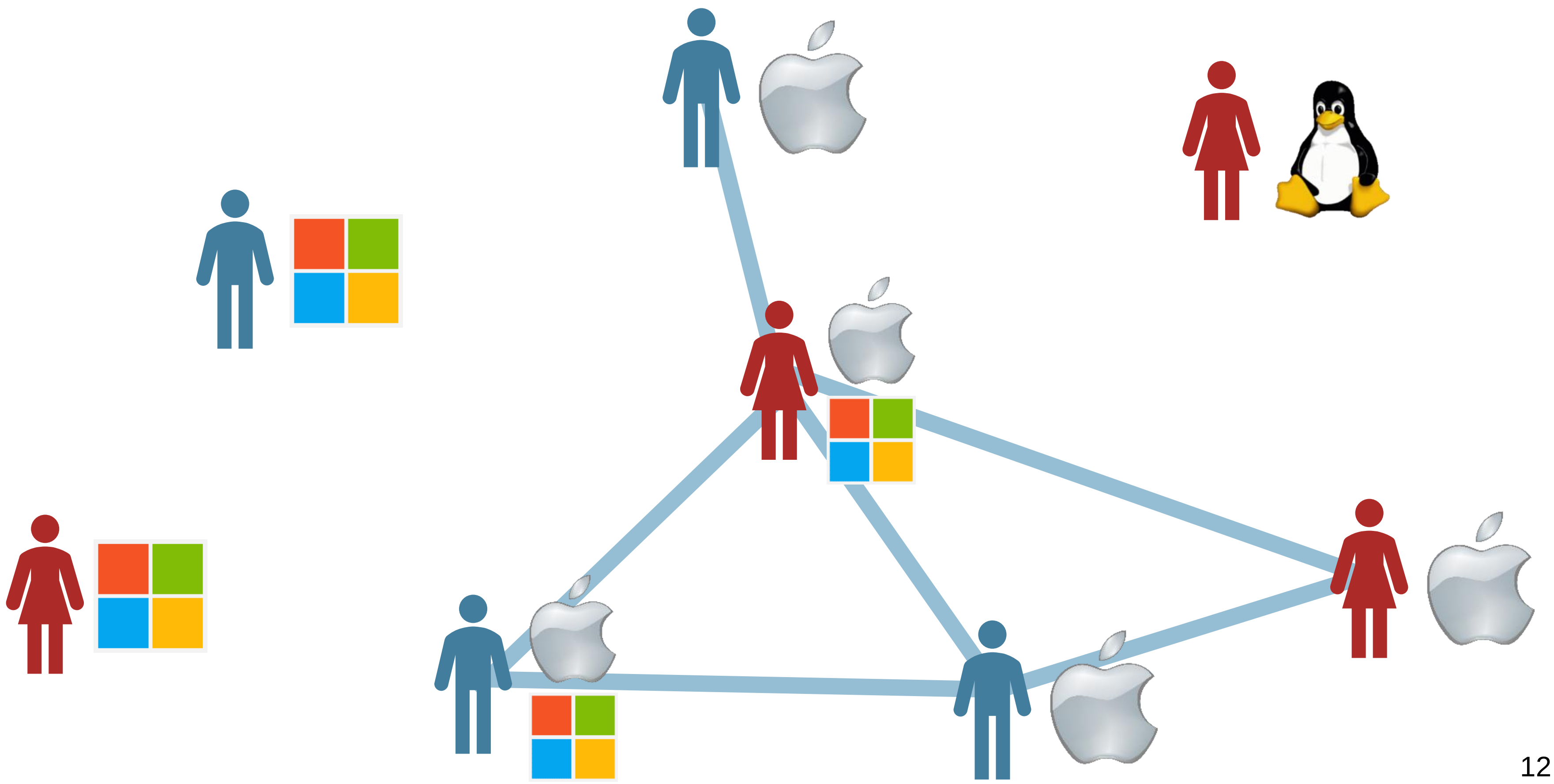

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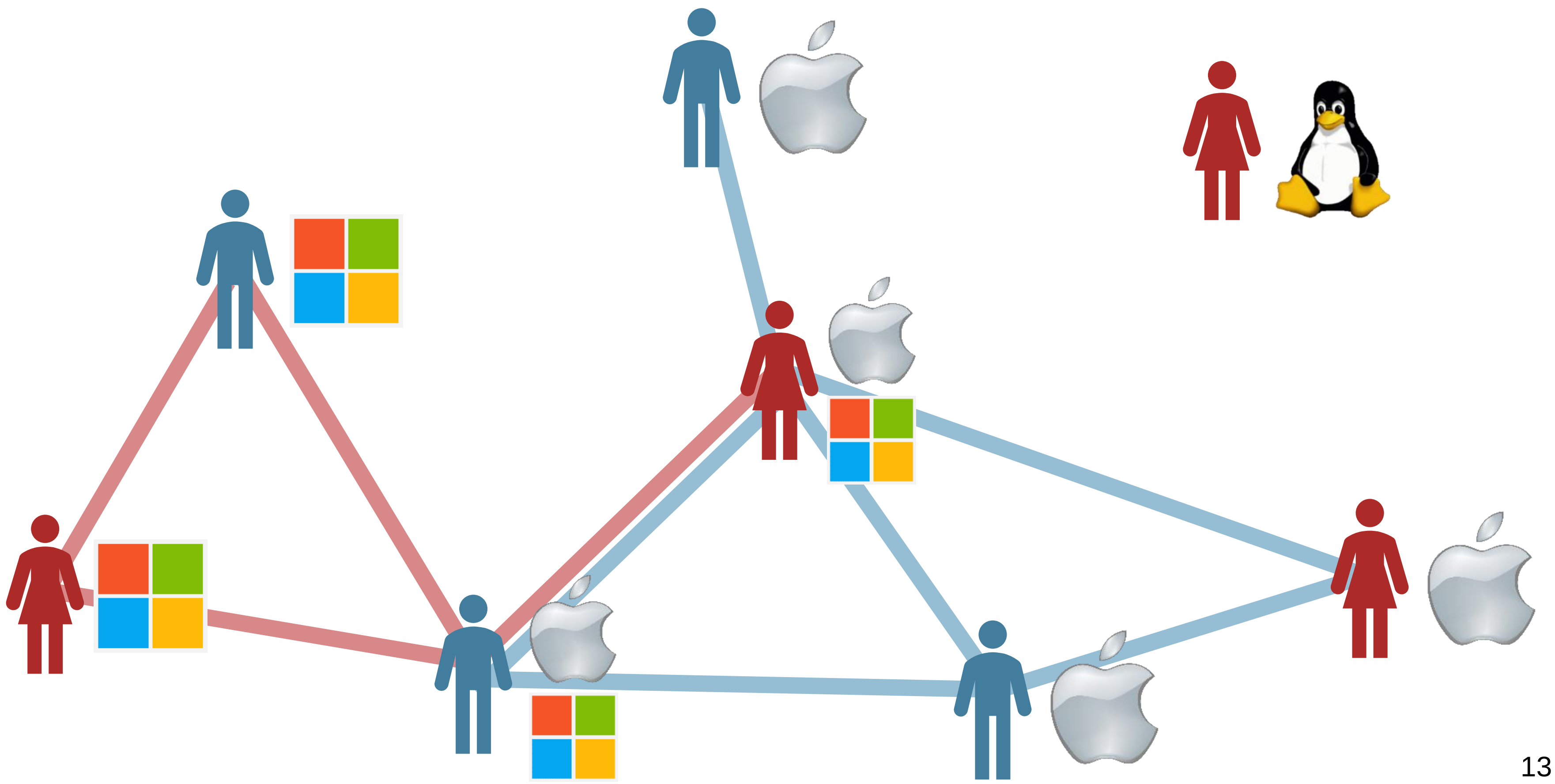

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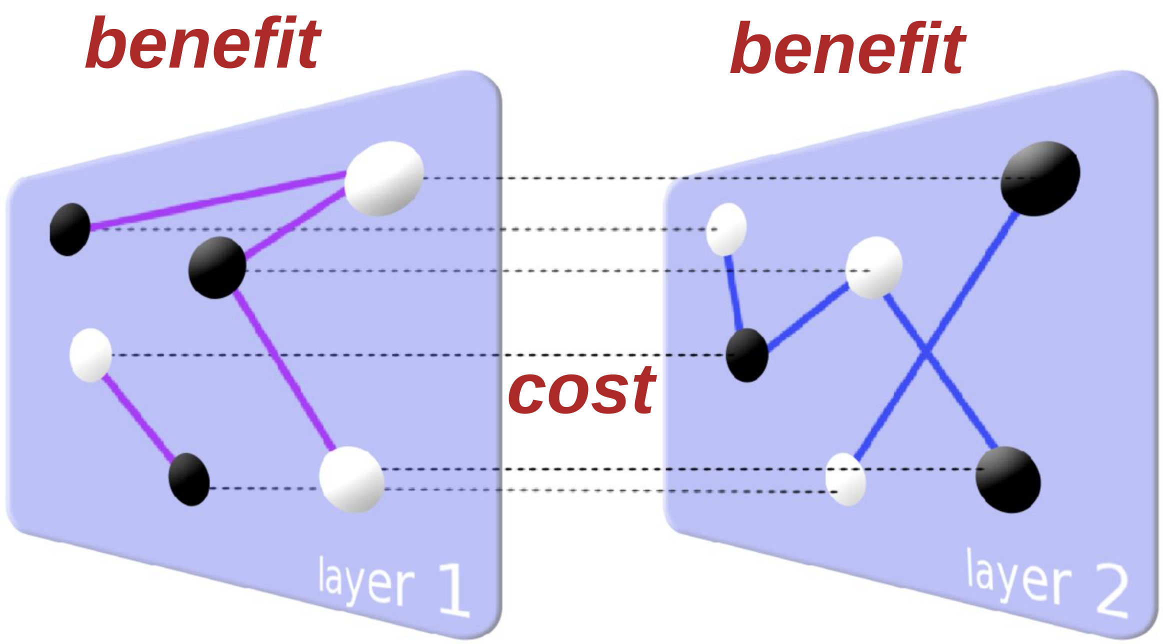

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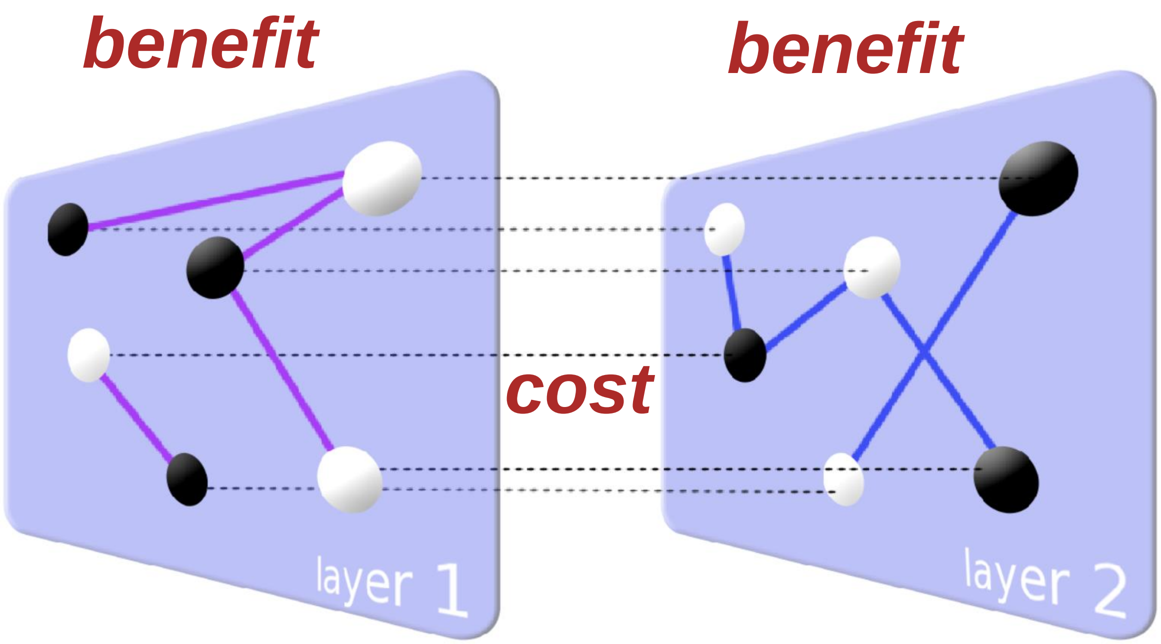
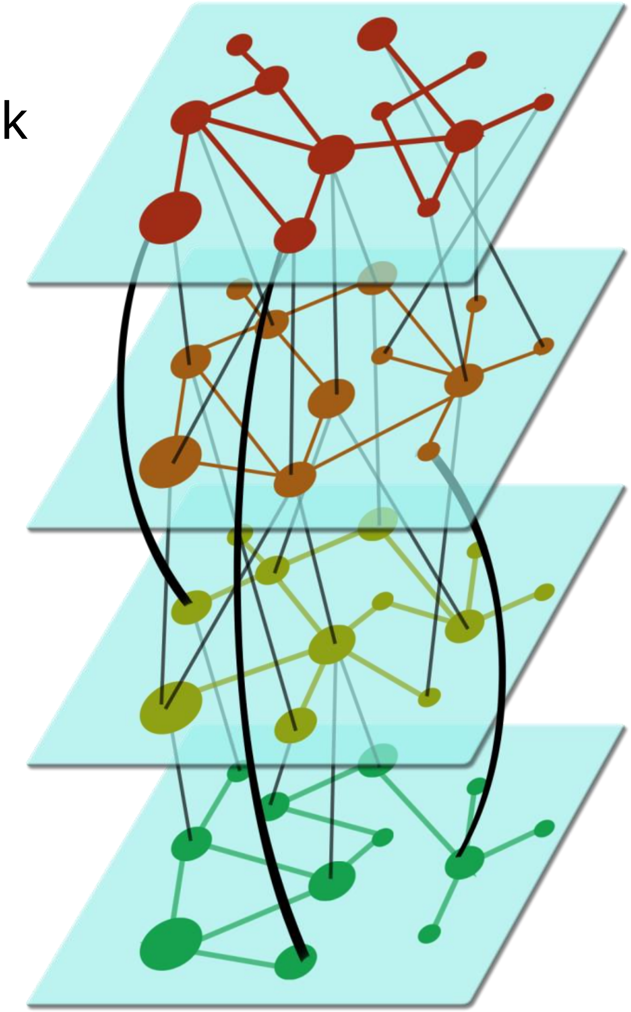


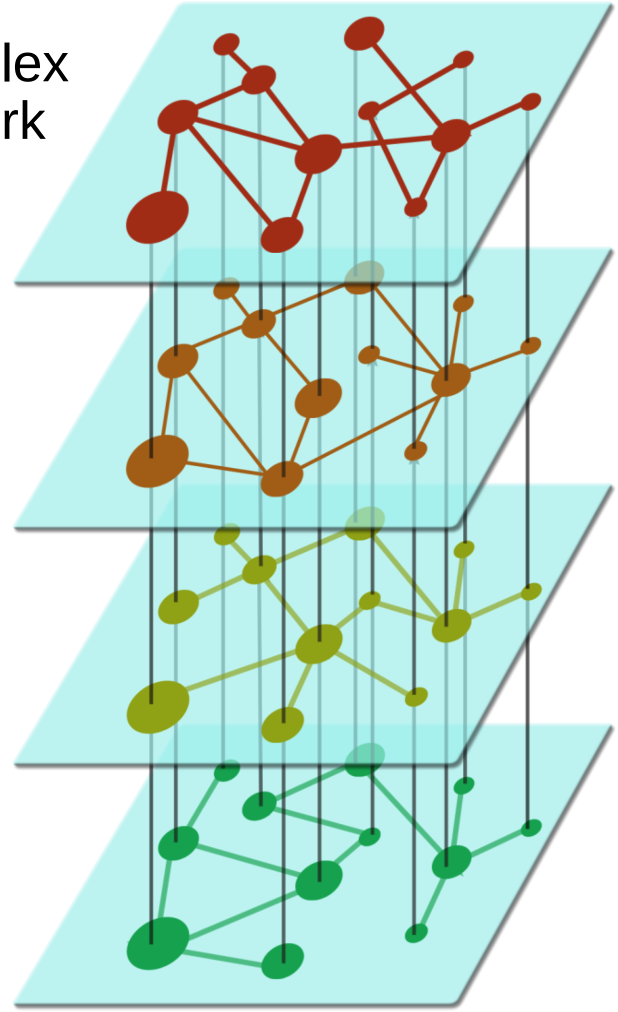
Competition between layers

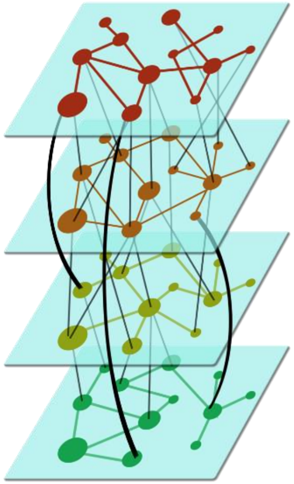
Structure

Interconnected multilayer network

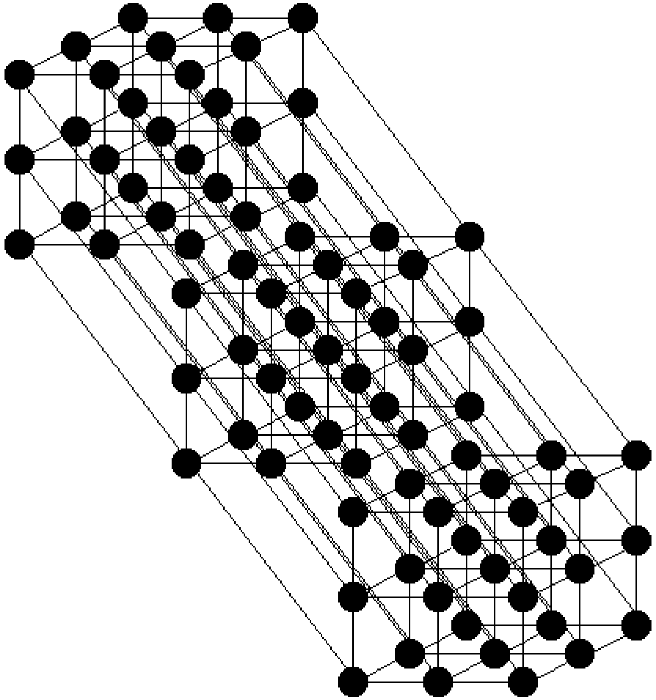


Multiplex network



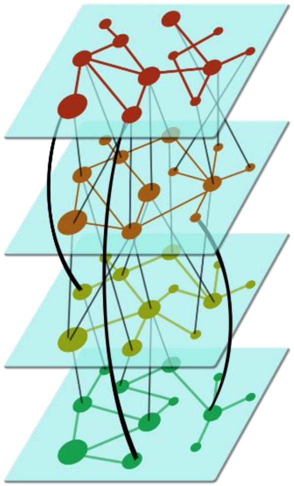


- Mathematical description
  - Adjacency (or weights) tensor
    - $M^{i\alpha}_{j\beta}$ node *i* in layer α connects to node *j* in layer β



$\mathbb{R}^{N\times L\times N\times L}$

De Domenico et al: Mathematical formulation of multilayer networks
Physical Review X **3** (2013) 041022

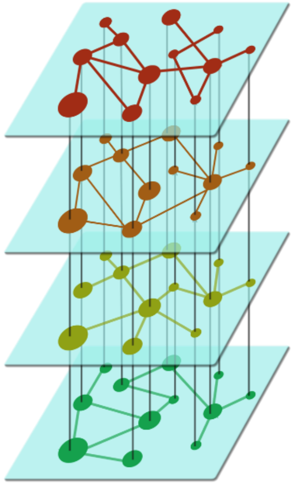


- Mathematical description
  - Supra-adjacency matrix
    - $\mathbf{W}^{(\alpha)}$ adjacency (or weights) matrix of layer α
    - $\mathbf{D}^{(\alpha\beta)}$ interaction matrix between layers α and β

$$\mathcal{J} = \begin{pmatrix} \mathbf{W}^{(1)} & 0 & \dots & 0 \\ 0 & \mathbf{W}^{(2)} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \mathbf{W}^{(L)} \end{pmatrix} + \begin{pmatrix} \mathbf{D}^{(11)} & \mathbf{D}^{(12)} & \dots & \mathbf{D}^{(1L)} \\ \mathbf{D}^{(21)} & \mathbf{D}^{(22)} & \dots & \mathbf{D}^{(2L)} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{D}^{(L1)} & \mathbf{D}^{(L2)} & \dots & \mathbf{D}^{(LL)} \end{pmatrix}$$

$$\mathcal{J} = \bigoplus_{\alpha=1}^{L} \mathbf{W}^{(\alpha)} + \mathcal{D}$$

Solé-Ribalta et al: Spectral properties of the Laplacian of multiplex networks
Physical Review E **88** (2013) 032807



- Mathematical description
  - Supra-adjacency matrix ***multiplex network***
    - $\mathbf{W}^{(\alpha)}$ adjacency (or weights) matrix of layer α
    - $D^{(\alpha\beta)}$ interaction strength between layers α and β

$$\mathcal{J} = \begin{pmatrix} \mathbf{W}^{(1)} & 0 & \dots & 0 \\ 0 & \mathbf{W}^{(2)} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \mathbf{W}^{(L)} \end{pmatrix} + \begin{pmatrix} D^{(11)}\mathbf{I} & D^{(12)}\mathbf{I} & \dots & D^{(1L)}\mathbf{I} \\ D^{(21)}\mathbf{I} & D^{(22)}\mathbf{I} & \dots & D^{(2L)}\mathbf{I} \\ \vdots & \vdots & \ddots & \vdots \\ D^{(L1)}\mathbf{I} & D^{(L2)}\mathbf{I} & \dots & D^{(LL)}\mathbf{I} \end{pmatrix}$$

$$\mathcal{J} = \bigoplus_{\alpha=1}^{L} \mathbf{W}^{(\alpha)} + \mathbf{D} \otimes \mathbf{I}$$

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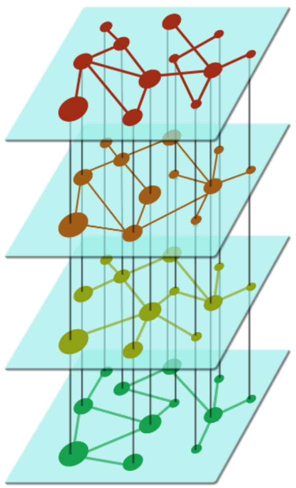

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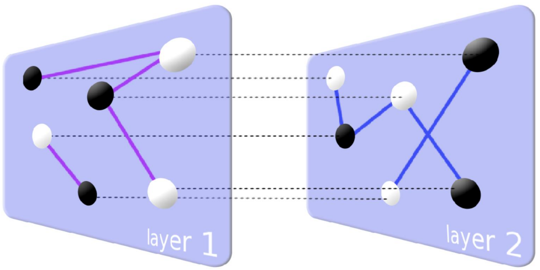
- Hypotheses
  - All nodes same interlayer strength $\mathbf{D}^{(\alpha\beta)} = D^{(\alpha\beta)}\mathbf{I}$
  - No self-loops $D^{(\alpha\alpha)} = 0$
  - Symmetry $D^{(\alpha\beta)} = D^{(\beta\alpha)}$

Solé-Ribalta et al: Spectral properties of the Laplacian of multiplex networks
Physical Review E **88** (2013) 032807

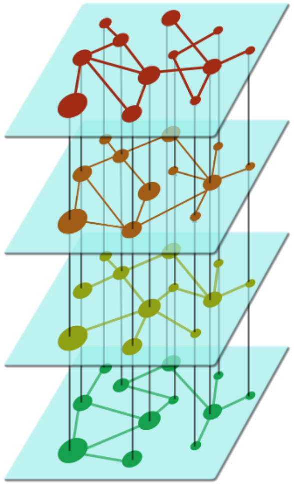


- Mathematical description
  - Two-layer multiplex networks

$$\mathcal{J} = \begin{pmatrix} \mathbf{W}^{(1)} & 0 \\ 0 & \mathbf{W}^{(2)} \end{pmatrix} + J_x \begin{pmatrix} 0 & \mathbf{I} \\ \mathbf{I} & 0 \end{pmatrix}$$

Gómez et al: Diffusion dynamics on multiplex networks
Physical Review Letters **110** (2013) 028701

Related dynamics



- Diffusion in multiplex networks

$$\dot{x}_i^{(\alpha)} = \sum_{j=1}^{N} w_{ij}^{(\alpha)} (x_j^{(\alpha)} - x_i^{(\alpha)}) + \sum_{\beta=1}^{L} D^{(\alpha\beta)} (x_i^{(\beta)} - x_i^{(\alpha)})$$

Laplacian

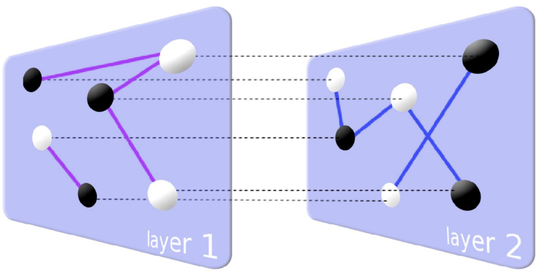
$$\mathcal{L} = \mathcal{L}^L + \mathcal{L}^I$$

$$\mathcal{L}^L = \bigoplus_{\alpha=1}^{M} L^{(\alpha)}$$

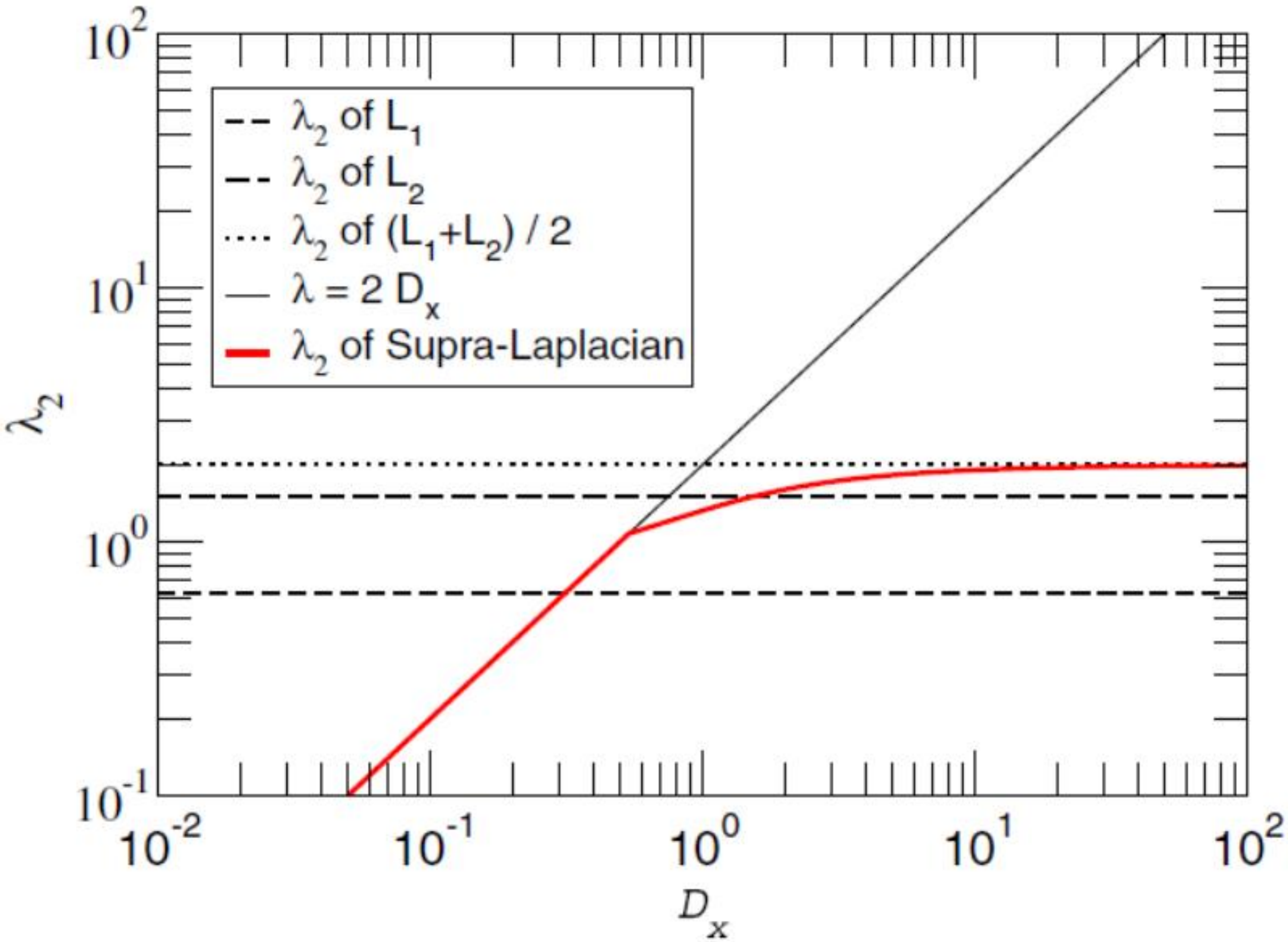
$$\mathcal{L}^I = L^I \otimes I$$

Diffusion time

$$\tau \sim \frac{1}{\lambda_2(\mathcal{L})}$$



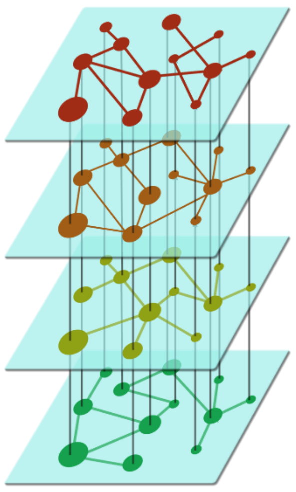
- Results
  - Superdiffusion! $\tau < \tau_1 < \tau_2$



$$L = 2$$

$$D_x \equiv D^{(12)}$$

Gómez et al: Diffusion dynamics on multiplex networks
Physical Review Letters **110** (2013) 028701



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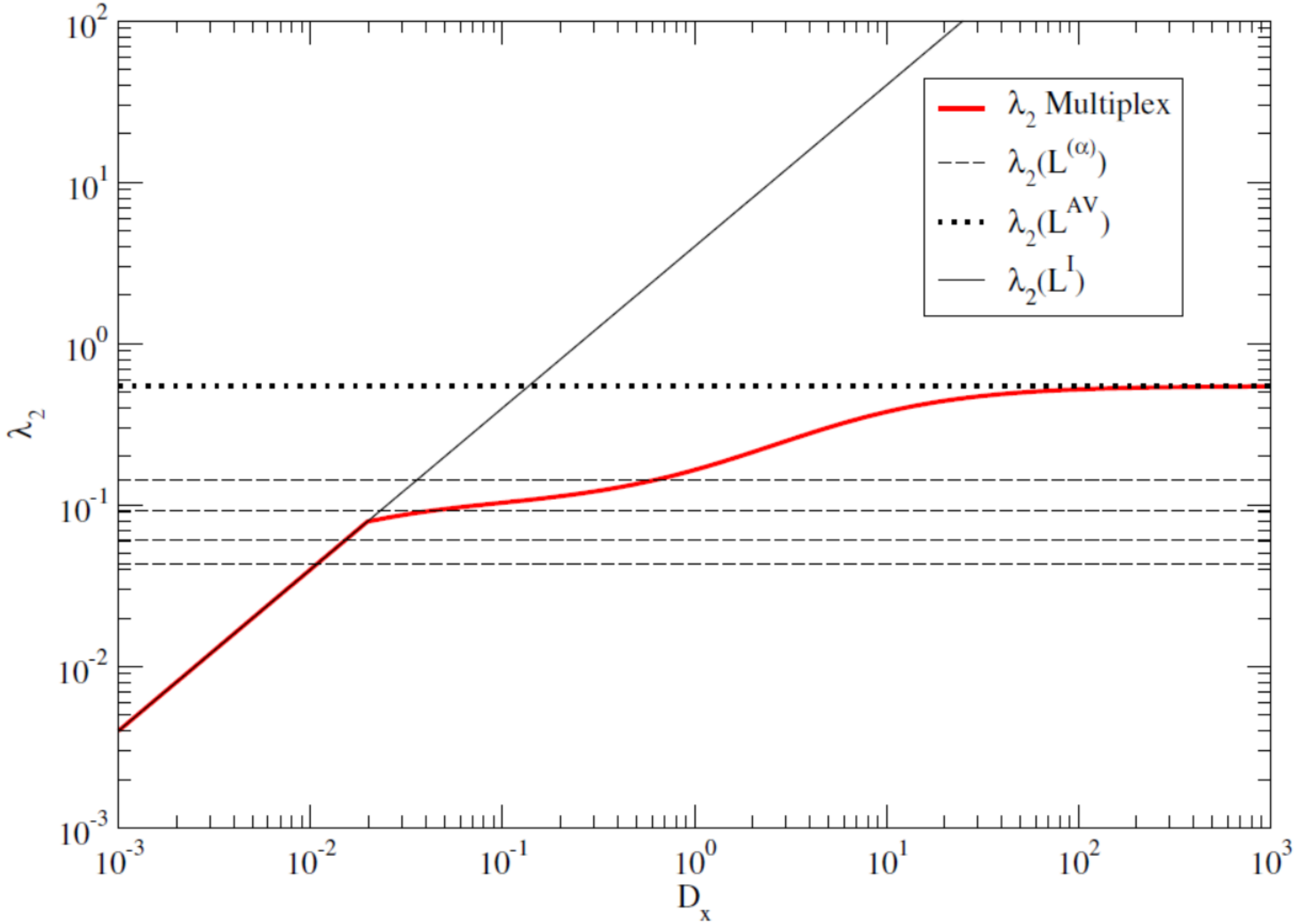


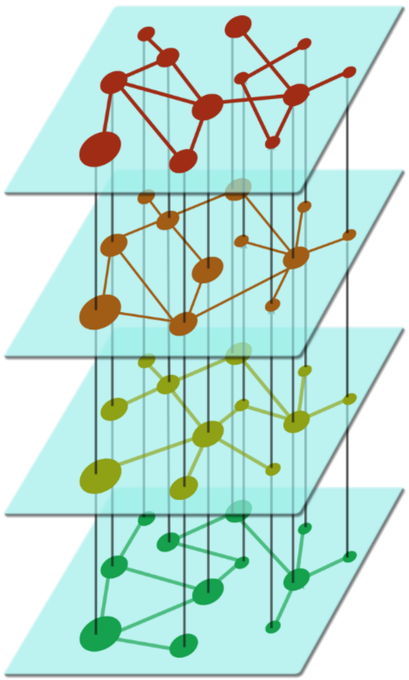
$$L = 4$$

$$D_x \equiv D^{(\alpha\beta)}$$

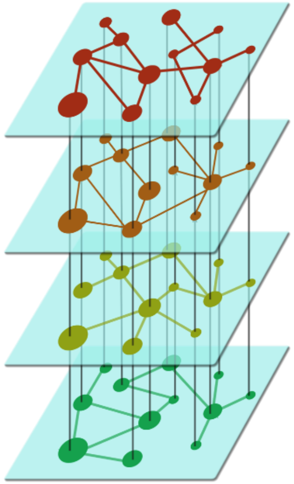
Solé-Ribalta et al: Spectral properties of the Laplacian of multiplex networks
Physical Review E **88** (2013) 032807

Competition dynamics

- Variables
  - $p_i^{(\alpha)}$ probability of node *i* being active in layer α



$$\sum_{\alpha=1}^{L} p_i^{(\alpha)} = 1$$



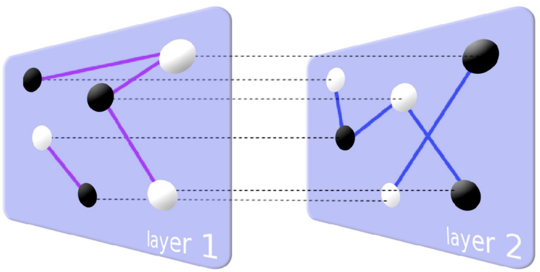
- **Competition**
  - Hamiltonian

$$H(\mathbf{P}) = -\sum_{\alpha,\beta=1}^{L}\sum_{i,j=1}^{N} J_{ij}^{(\alpha\beta)} p_i^{(\alpha)} p_j^{(\beta)}$$

where

$$\mathcal{J} = \bigoplus_{\alpha=1}^{L} \mathbf{W}^{(\alpha)} + \mathbf{D} \otimes \mathbf{I}$$

Gómez-Gardeñes et al: Layer-layer competition in multiplex complex networks
Philosophical Transactions of the Royal Society A **373** (2015) 20150117



- Competition in two-layer multiplex
  - Variables

$$p_i^{(1)} = p_i \qquad\qquad p_i^{(2)} = 1 - p_i$$

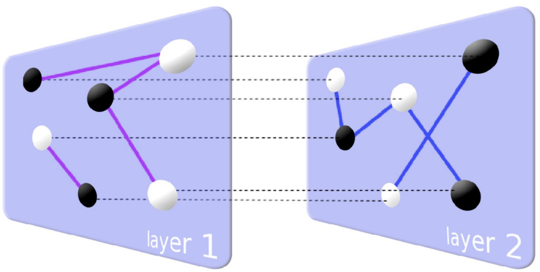
  - Hamiltonian

$$H(\vec{p}) = -\sum_{i,j=1}^{N} W_{ij}^{(1)} p_i p_j - \sum_{i,j=1}^{N} W_{ij}^{(2)} (1-p_i)(1-p_j)$$

$$-2J_x \sum_{i=1}^{N} p_i(1-p_i)$$

Gómez-Gardeñes et al: Layer-layer competition in multiplex complex networks
Philosophical Transactions of the Royal Society A **373** (2015) 20150117

- Competition in two-layer multiplex



$$H(\vec{p}) = -\sum_{i,j=1}^{N} W_{ij}^{(1)} p_i p_j - \sum_{i,j=1}^{N} W_{ij}^{(2)} (1-p_i)(1-p_j) - 2J_x \sum_{i=1}^{N} p_i(1-p_i)$$

Gómez-Gardeñes et al: Layer-layer competition in multiplex complex networks
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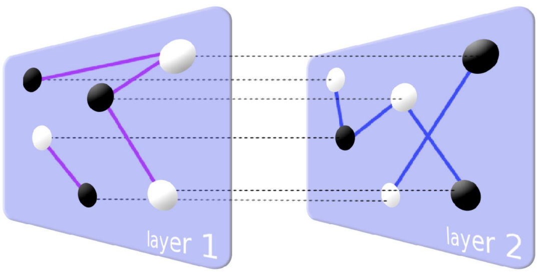
- Competition in two-layer multiplex



$$H(\vec{p}) = \boxed{-\sum_{i,j=1}^{N} W_{ij}^{(1)} p_i p_j} - \sum_{i,j=1}^{N} W_{ij}^{(2)} (1-p_i)(1-p_j) - 2J_x \sum_{i=1}^{N} p_i(1-p_i)$$

Minimum value when
all $p_i = 1$

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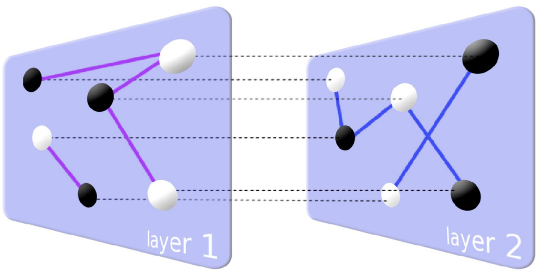
- Competition in two-layer multiplex



$$H(\vec{p}) = -\sum_{i,j=1}^{N} W_{ij}^{(1)} p_i p_j \boxed{-\sum_{i,j=1}^{N} W_{ij}^{(2)} (1-p_i)(1-p_j)} - 2J_x \sum_{i=1}^{N} p_i (1-p_i)$$

Minimum value when
all $p_i = 0$

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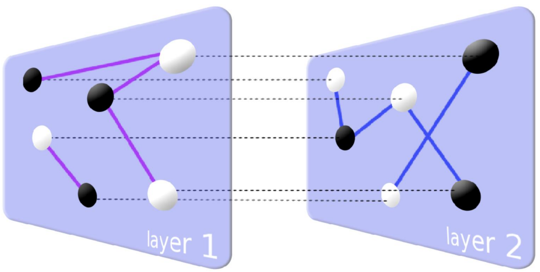
- Competition in two-layer multiplex



$$H(\vec{p}) = -\sum_{i,j=1}^{N} W_{ij}^{(1)} p_i p_j - \sum_{i,j=1}^{N} W_{ij}^{(2)} (1-p_i)(1-p_j) \boxed{- 2J_x \sum_{i=1}^{N} p_i (1-p_i)}$$

↓

Minimum value when all $p_i = 0.5$

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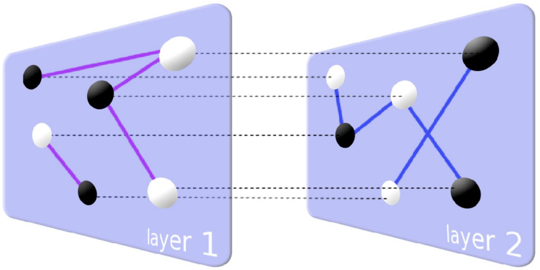

- Magnetization

$$M(\vec{p}) = \frac{1}{N}\sum_{i=1}^{N}(2p_i - 1)$$

- All $p_i = 1$ ➔ $M = +1$ ➔ All nodes in first layer
- All $p_i = 0.5$ ➔ $M = 0$ ➔ All nodes equally in all layers
- All $p_i = 0$ ➔ $M = -1$ ➔ All nodes in second layer

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Ground state

- Minimize

$$H(\vec{p}) = -\sum_{i,j=1}^{N} W_{ij}^{(1)} p_i p_j - \sum_{i,j=1}^{N} W_{ij}^{(2)} (1-p_i)(1-p_j) - 2J_x \sum_{i=1}^{N} p_i(1-p_i)$$

with the constraints

$0 \leqslant p_i \leqslant 1$ ➡ solution inside the $[0,1]^N$ hypercube

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Ground state

Quadratic programming problem

- Minimize

$$H(\vec{p}) = -\sum_{i,j=1}^{N} W_{ij}^{(1)} p_i p_j - \sum_{i,j=1}^{N} W_{ij}^{(2)} (1-p_i)(1-p_j)$$

$$-2J_x \sum_{i=1}^{N} p_i (1-p_i)$$

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$0 \leqslant p_i \leqslant 1$ ➡ solution inside the $[0,1]^N$ hypercube

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- Gradient

$$\frac{\partial H}{\partial p_i} = -2\sum_{j=1}^{N} W_{ij}^{(1)} p_j + 2\sum_{j=1}^{N} W_{ij}^{(2)}(1-p_j) - 2J_x(1-2p_i)$$

- Zero gradient equation

$$\left[2J_x\mathbf{I} - \left(\mathbf{W}^{(1)} + \mathbf{W}^{(2)}\right)\right]\vec{p} = J_x\vec{1} - \vec{s}^{(2)} \quad \Rightarrow \quad \vec{p}^{\star}$$

- Hessian

$$\frac{\partial^2 H}{\partial p_i \partial p_j} = 2\left(2J_x\delta_{ij} - W_{ij}^{(1)} - W_{ij}^{(2)}\right)$$

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- Ground state conditions
  - If $\vec{p}^{\star}$ inside $[0,1]^N$ and Hessian positive definite
    - $\vec{p}^{\star}$ is feasible solution
    - $\vec{p}^{\star}$ is the ground state
  - Else
    - Ground state lies in one side of the hypercube $[0,1]^N$
    - If Hessian not positive definite ➡ **NP-hard** problem

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- Asymptotic limits
  - When $J_x \gg 1$
  - When $J_x = 0$

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- Asymptotic limits
  - When $J_x \gg 1$

$$H(\vec{p}) = -\sum_{i,j=1}^{N} W_{ij}^{(1)} p_i p_j - \sum_{i,j=1}^{N} W_{ij}^{(2)} (1-p_i)(1-p_j) \boxed{- 2J_x \sum_{i=1}^{N} p_i (1-p_i)}$$

Other terms negligible

Gómez-Gardeñes et al: Layer-layer competition in multiplex complex networks
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- Asymptotic limits
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$\boxed{M = 0}$ ⟵ Minimum value when all $p_i = 0.5$ ⟵ Other terms negligible

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- Asymptotic limits
  - When $J_x = 0$

$$H(\vec{p}) = \boxed{-\sum_{i,j=1}^{N} W_{ij}^{(1)} p_i p_j - \sum_{i,j=1}^{N} W_{ij}^{(2)} (1-p_i)(1-p_j)} - 2J_x \sum_{i=1}^{N} p_i(1-p_i)$$

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- Asymptotic limits
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$$s^{(\alpha)} = \sum_{i,j=1}^{N} W_{ij}^{(\alpha)}$$

If $s^{(1)} > s^{(2)}$ ➡ $\boxed{M = +1}$

If $s^{(1)} < s^{(2)}$ ➡ $\boxed{M = -1}$

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- Asymptotic limits

$J_x = 0$ $J_x \gg 1$

Localized activity in first layer

Mixed activity in all layers

(supposing $s^{(1)} > s^{(2)}$)

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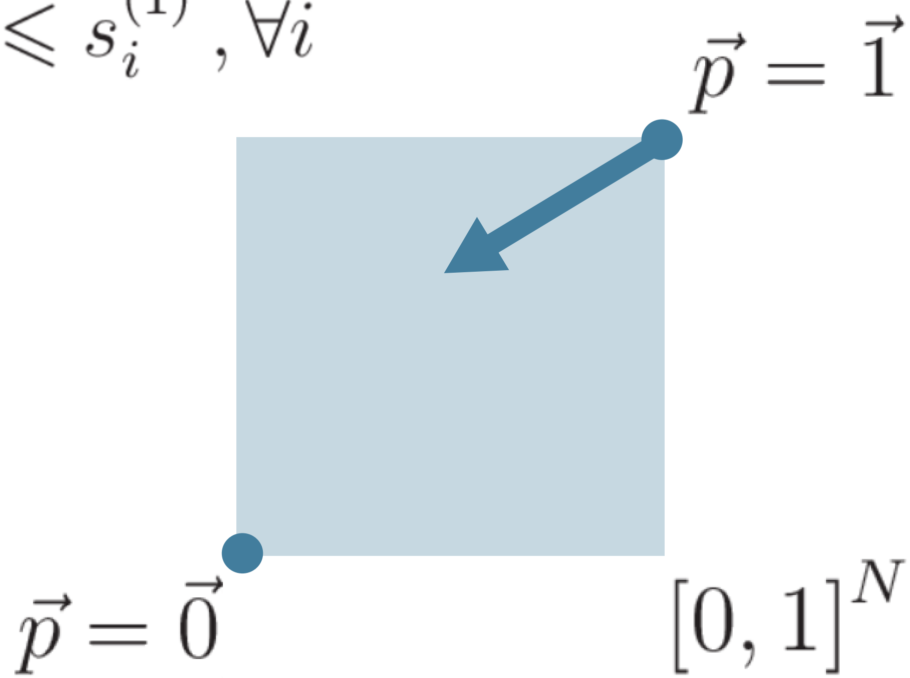
- Gradient at $\vec{p} = \vec{1}$

$$\left.\frac{\partial H}{\partial p_i}\right|_{\vec{p}=\vec{1}} = 2\left(J_x - s_i^{(1)}\right)$$

  - If $J_x \leqslant s_i^{(1)}\ , \forall i$



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- Gradient at $\vec{p} = \vec{1}$

$$\left.\frac{\partial H}{\partial p_i}\right|_{\vec{p}=\vec{1}} = 2\left(J_x - s_i^{(1)}\right)$$

  - If J_x below $J_x^c = \min_{i=1,\ldots,N}(s_i^{(1)}) = s_{\min}^{(1)}$
    - $\vec{p}^\star = \vec{1}$

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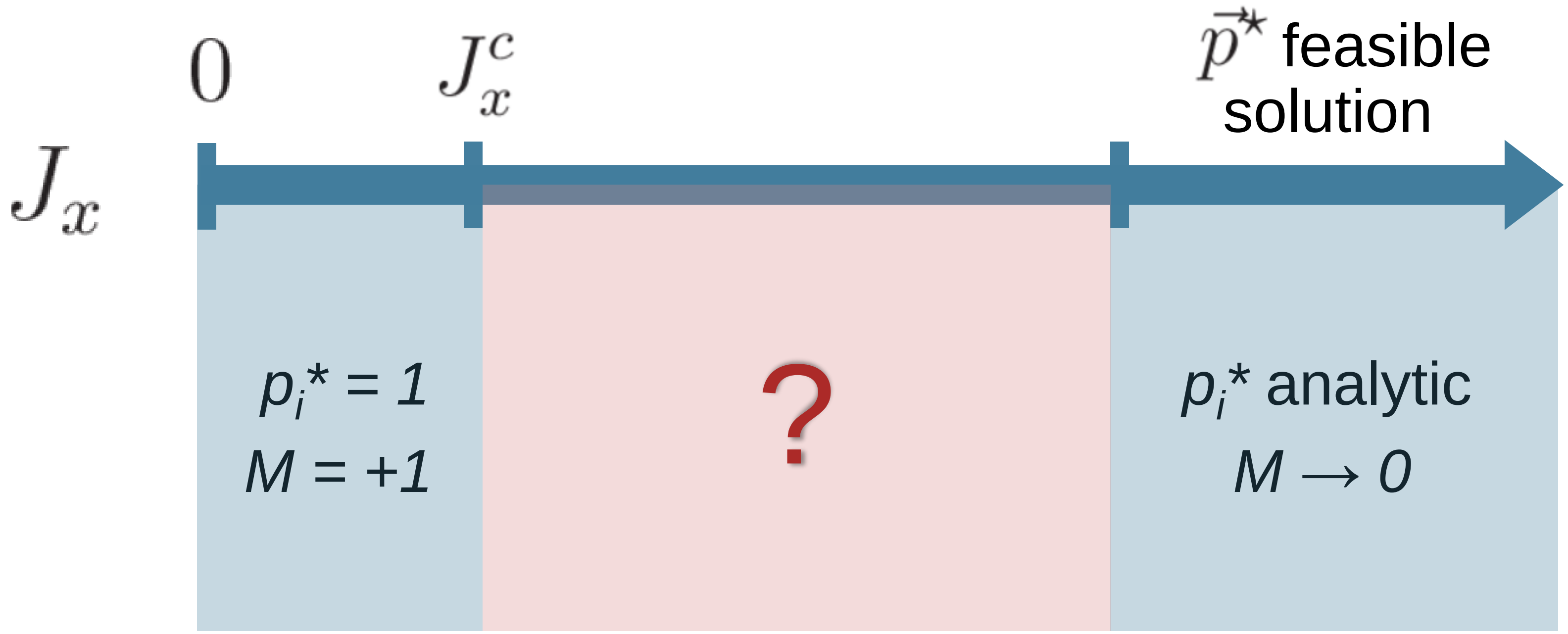
- Solutions diagram



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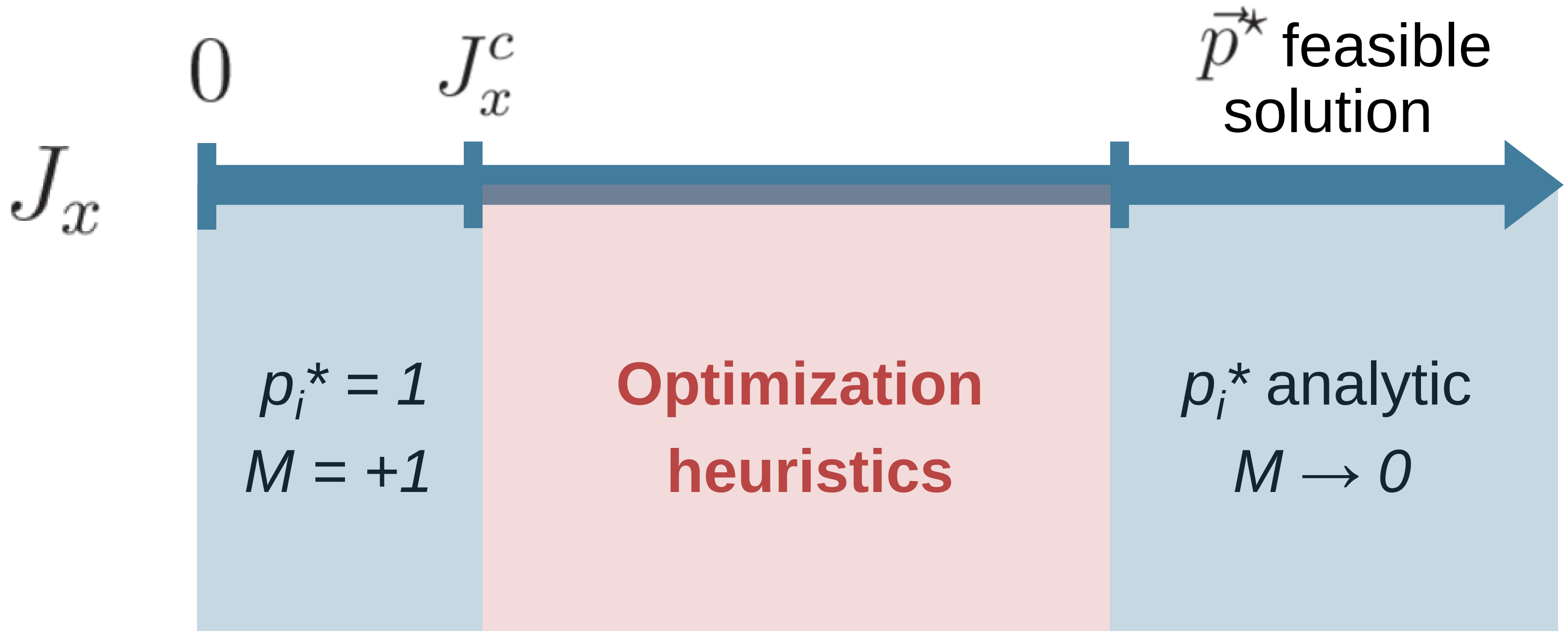
- Solutions diagram



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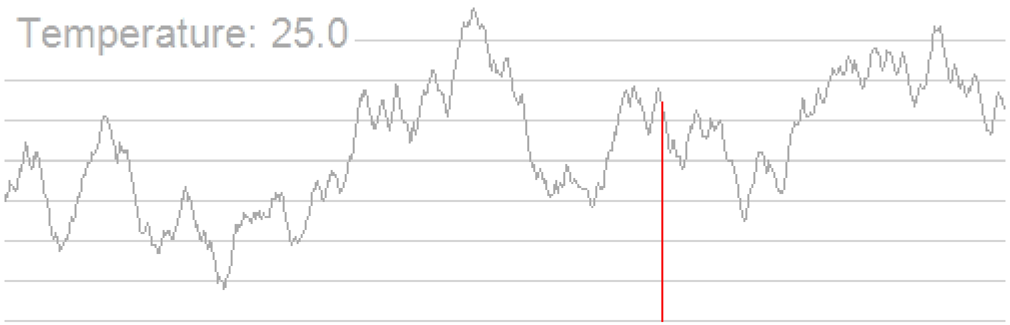
Combinatorial optimization

- NP-complete / NP-hard optimization problems
  - Many variables
  - Huge search space
  - No known polynomial time algorithms
- Algorithms
  - Local search
  - Collective search
  - Hybrid search

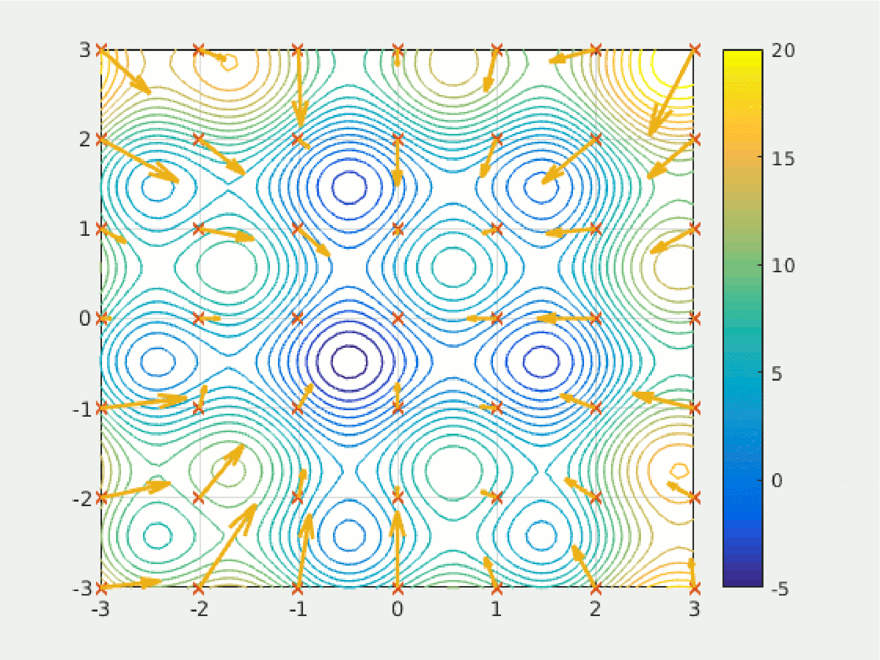
- Local search
  - Characteristics
    - One individual moves in the search state
    - Travel guided by local information
    - Short-term memory
    - Try to avoid local optima
  - Some methods
    - Gradient descent
    - Simulated annealing
    - Tabu search
    - Extremal optimization



- Local search methods
  - Gradient descent
    - Continuous variables
    - Needs the gradient
    - Easily stacked in local minima
    - Add noise or inertia to improve search
  - Simulated annealing
    - Inspired by physics at equilibrium
    - Adequate for discrete variables
    - Explore neighbors
    - Allow uphill moves with certain probability (temperature)

- Local search methods
  - Tabu search
    - Adequate for discrete variables
    - Explore neighbors
    - Forbid uphill moves in a certain tabu list
  - Extremal optimization
    - Inspired by physics out of equilibrium
    - Adequate for discrete variables
    - Explore neighbors
    - Objective function sum of one-variable terms
    - Improve the worst contribution

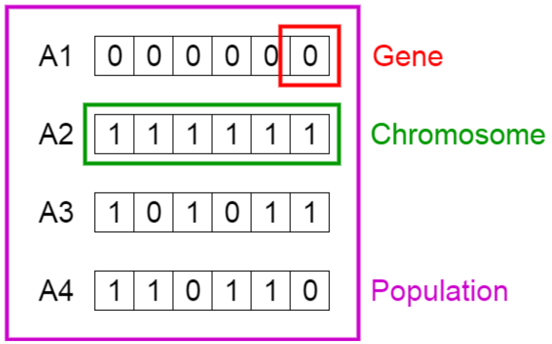
- Collective search
  - Characteristics
    - Several individuals move in the search state
    - Communication between individuals
    - Travel guided by local and global information
    - Long-term memory, swarm intelligence, diversity
  - Some methods
    - Evolutionary computation
      - Genetic algorithms
      - Evolution strategies
    - Swarm intelligence
      - Particle swarm optimization
      - Ant colony systems
      - Artificial bee colony

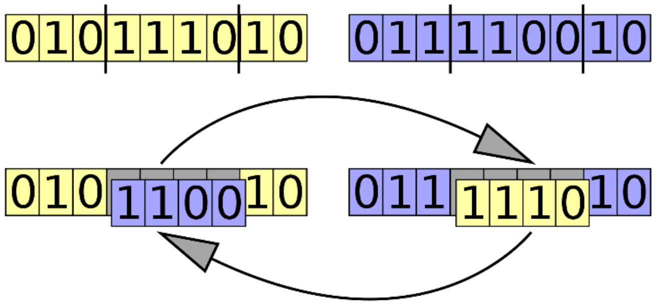


- Collective search methods
  - Genetic algorithms
    - Inspired by evolution and natural selection
    - Adequate for discrete binary variables

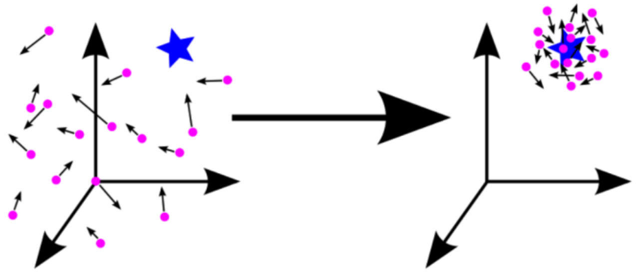
    - Population of individuals, each with a chromosome
    - Iteration over generations

    - Selection
    - Reproduction (crossover)
    - Mutation
    - Elitism



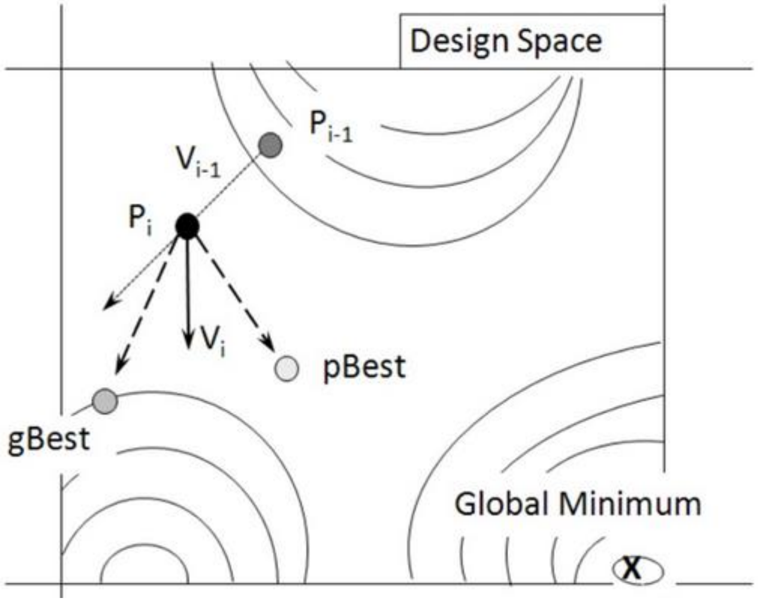


- Collective search methods
  - Particle swarm optimization (PSO)
    - Inspired by bird flocks and fish schools
    - Adequate for continuous variables



    - Set of particles
    - Each particle has position and velocity
    - Each particle remembers its best position

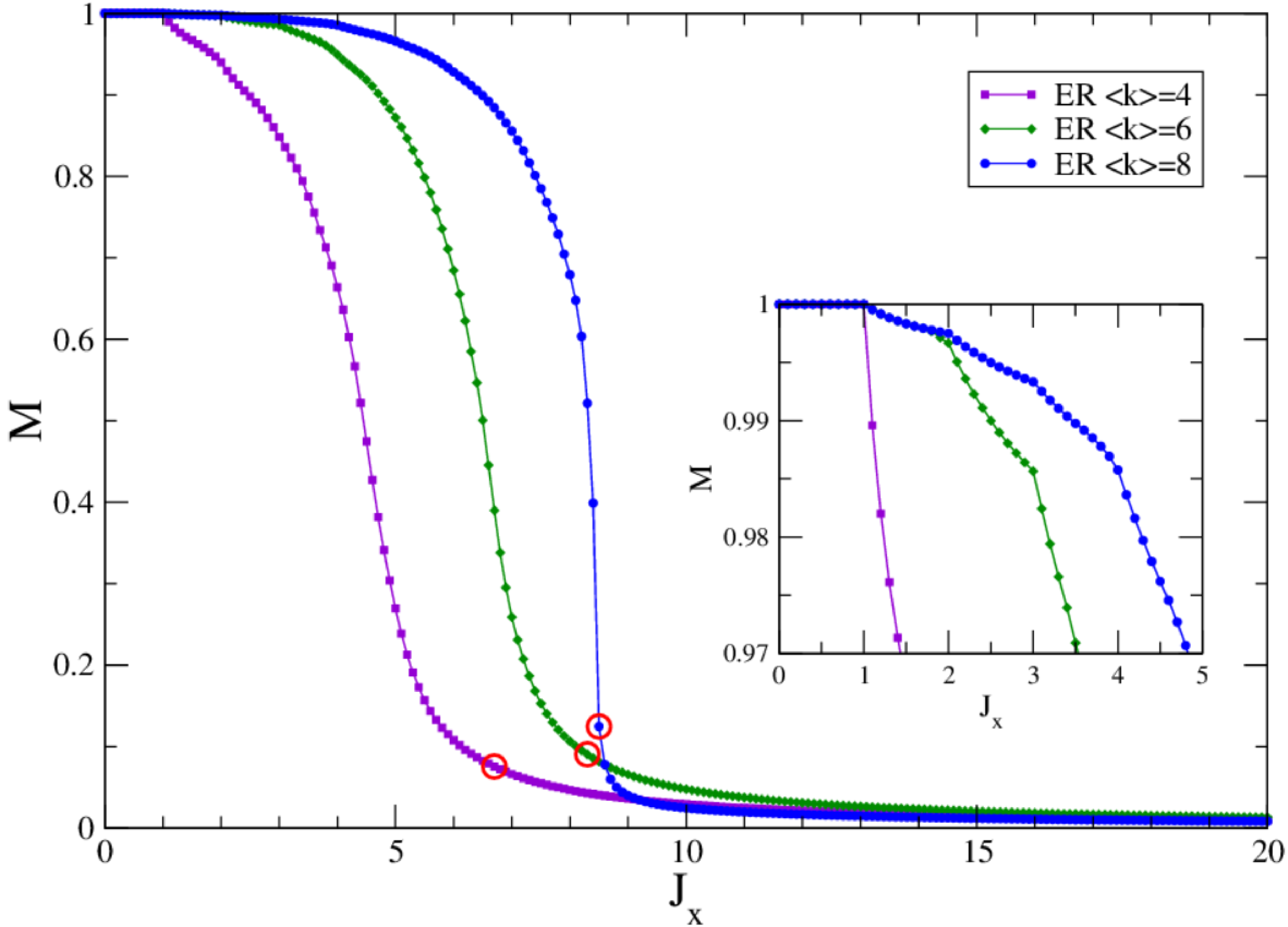
    - Inertia
    - Approach local best
    - Approach global best



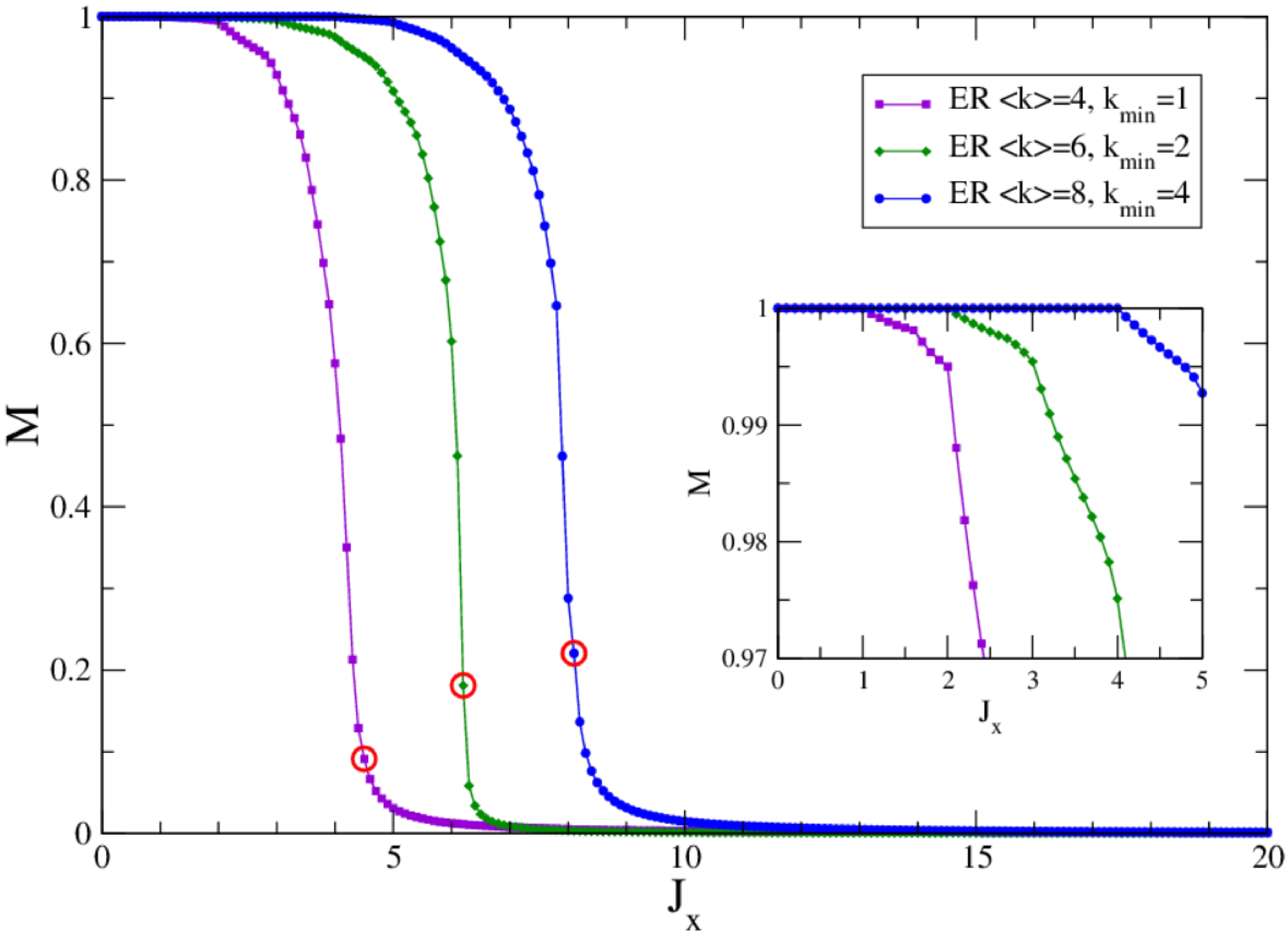
- Hybrid search
  - Collective search + Local optimization
  - Memetic algorithms

Results

- Ground state search
  - Standard optimization package
    - METIS, **failed**
  - Local search
    - Simulated annealing, **failed**
  - Collective search
    - Particle swarm optimization, **success** → selected



ER <k>=4
ER <k>=6
ER <k>=8
M
J x
0
0.2
0.4
0.6
0.8
1
5
10
15
20
0.97
0.98
0.99
2
3
4



1
0.8
0.6
0.4
0.2
0
M
0
5
10
15
20
J_x
ER <k>=4, k_{min}=1
ER <k>=6, k_{min}=2
ER <k>=8, k_{min}=4
1
0.99
0.98
0.97
M
0
1
2
3
4
5
J_x

Concluding remarks

- Model of competition between layers
- Analytically tractable in part of the phase diagram
- Optimization heuristics required
  - There is life beyond simulated annealing!
  - Use the most appropriate
  - Try several
  - Check always the natural candidate solutions

Thank you for your attention!

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- References
  - **J Gómez-Gardeñes**, **M De Domenico**, **G Gutiérrez**, **A Arenas**, S Gómez
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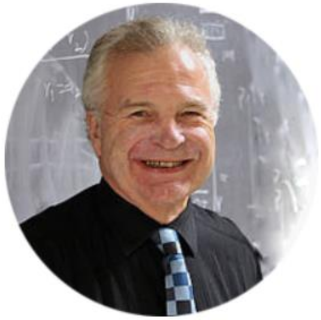

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