Multi-scale mixing in complex networks

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Assortativity



Disassortativity



Mixing in real networks



Mixing in real networks



Mixing in real networks



Generalisation, not rules!



$$r_{\rm global} = \frac{\sum_g e_{gg} - \sum_g a_g b_g}{1 - \sum_g a_g b_g}$$





Newman "Mixing patterns in networks" Phys. Rev. E (2003)





 $r \sim -0.3$

Assortativity is correlation across edges

Assortativity is correlation across edges



Anscombe, "Graphs in Statistical Analysis". American Statistician (1973)

All these networks have assortativity r=0



Peel, Delvenne, Lambiotte, "Multiscale mixing patterns in networks". PNAS (2018)

Can we measure assortativity locally?

Time series analysis



The mean is only representative of the data around the middle of the time series

Time series analysis



Exponentially weighted mean

Recent points are more relevant

$$S_t = lpha y_{t-1} + (1-lpha) S_{t-1} \quad 0 < lpha \leq 1$$

Random walks on a graph

 $e_{gh} = \sum_{ij} w(i) \quad \frac{A_{ij}}{k_i} \, \delta_{y_i,g} \delta_{y_j,h}$



g

 $w(i;\ell)$

stationary distribution of a random walk (PageRank)

Random walks on a graph

$$e_{gh}(\alpha,\ell) = \sum_{ij} w(i;\ell) \frac{A_{ij}}{k_i} \, \delta_{y_i,g} \delta_{y_j,h}$$



stationary distribution of a random walk with restart (Personalised PageRank)

Random walks on a graph

$$e_{gh}(\alpha, \ell) = \sum_{ij} w(i; \ell) \frac{A_{ij}}{k_i} \,\delta_{y_i, g} \delta_{y_j, h}$$
$$r(\alpha, \ell) = \frac{\sum_g e_{gg}(\alpha, \ell) - \sum_g a_g b_g}{1 - \sum_g a_g b_g}$$











Karate club - factions



Weddell Sea food web



Facebook 100 – residence



Peel, Delvenne, Lambiotte, "Multiscale mixing patterns in networks". PNAS (2018)

Summary

Assortativity is a correlation coefficient (or autocorrelation)

- can lead to ambiguity in its interpretation

Often we observe variation in the assortativity across the network

- In some cases its not possible to have constant assortativity

Our framework provides a means to compute global network measures within a local neighbourhood

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Questions?





Multiscale mixing patterns in networks

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