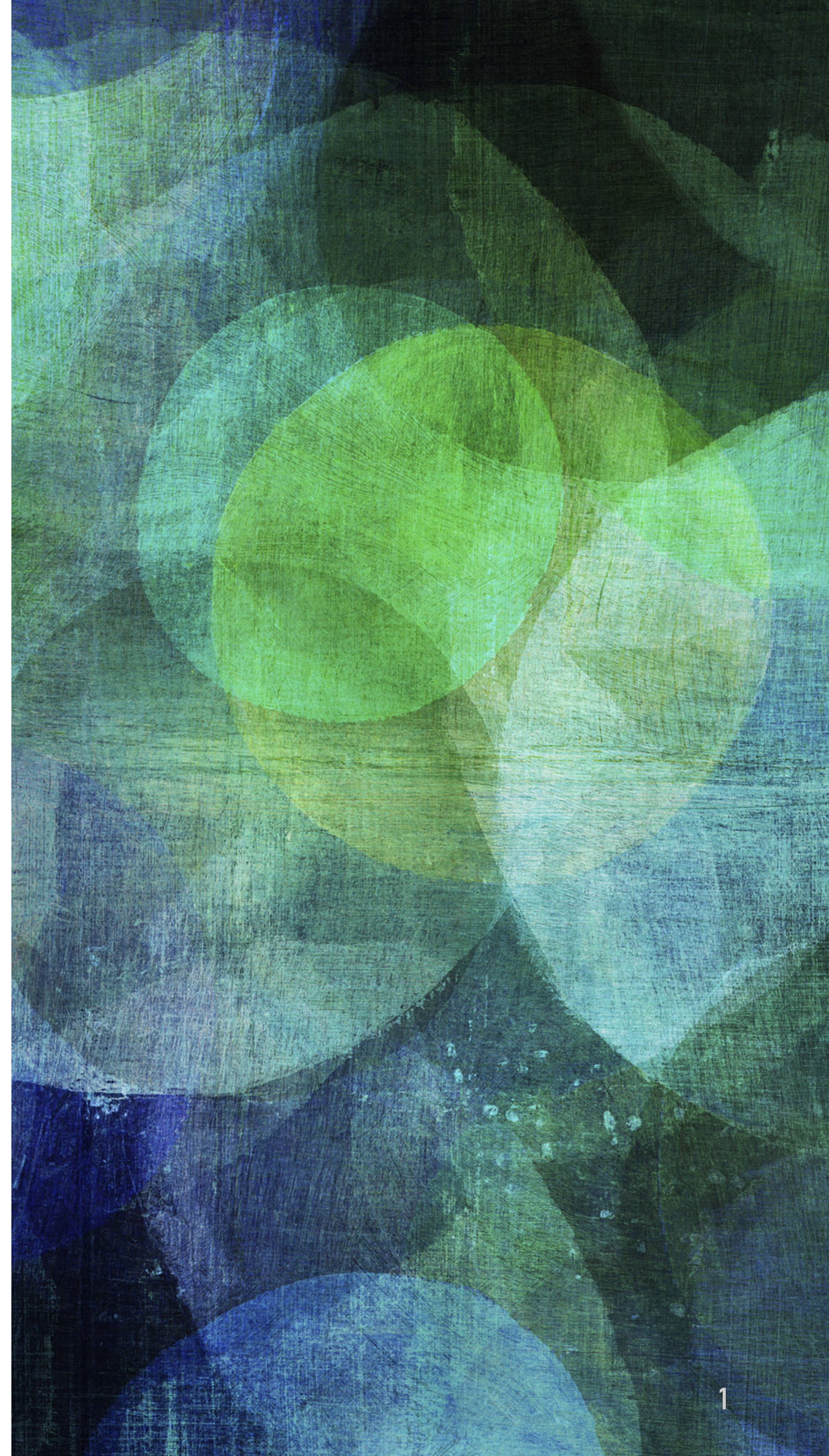
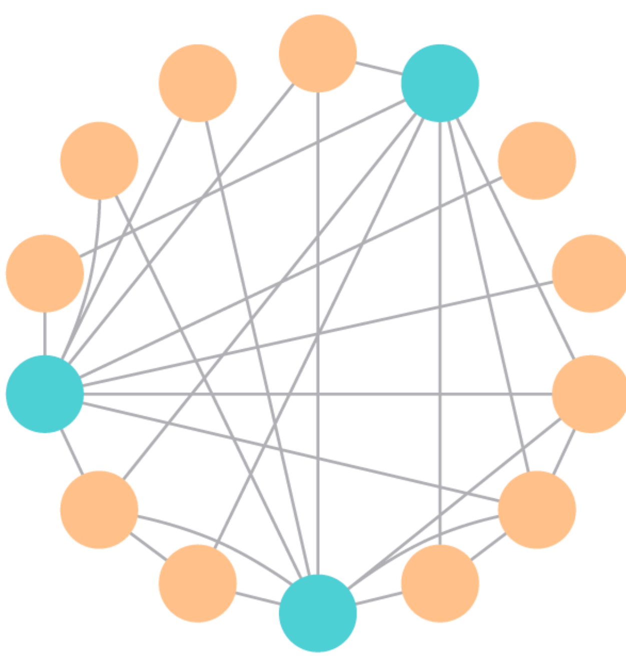
BIASED PERCEPTIONS IN DIRECTED NETWORKS

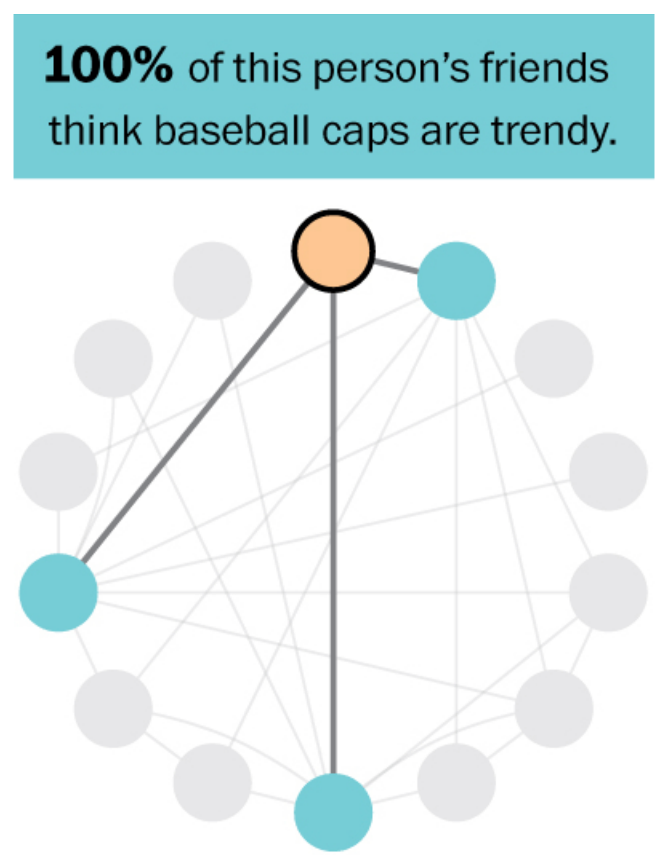
*Nazanin Alipourfard, Buddhika Netasinghe, **Andrés Abeliuk**, Vikram Krishnamurthy, Kristina Lerman*

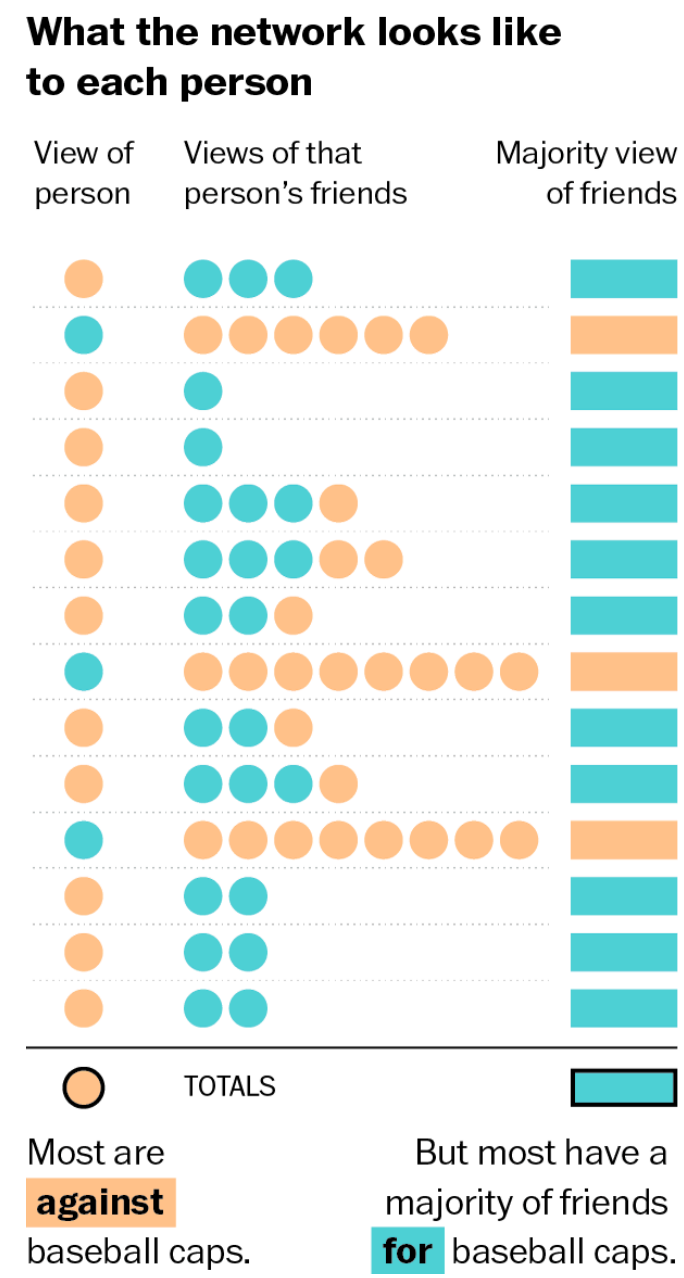


THE MAJORITY ILLUSION

- We see the world through our own personal lenses.
- Local knowledge, can lead to false conclusions.



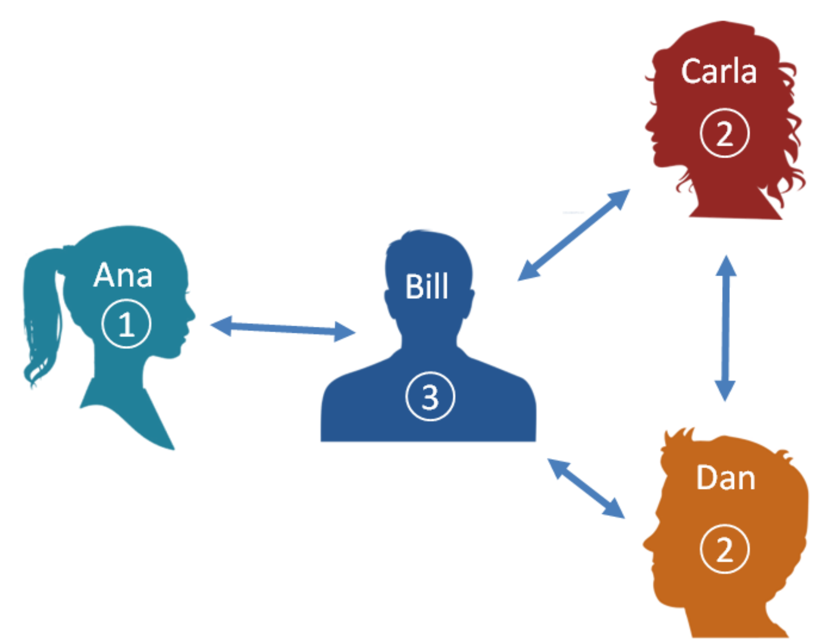




Kristina Lerman et al. The majority illusion in social networks. PloS one, 2016.

FRIENDSHIP PARADOX

- Your are **less popular** than your friends on average.
- **Any** trait correlated with popularity will create a bias:
  - Scientists tend to have **less impact** than their co-authors
  - People are **less happy** than their friends.



RESEARCH QUESTIONS

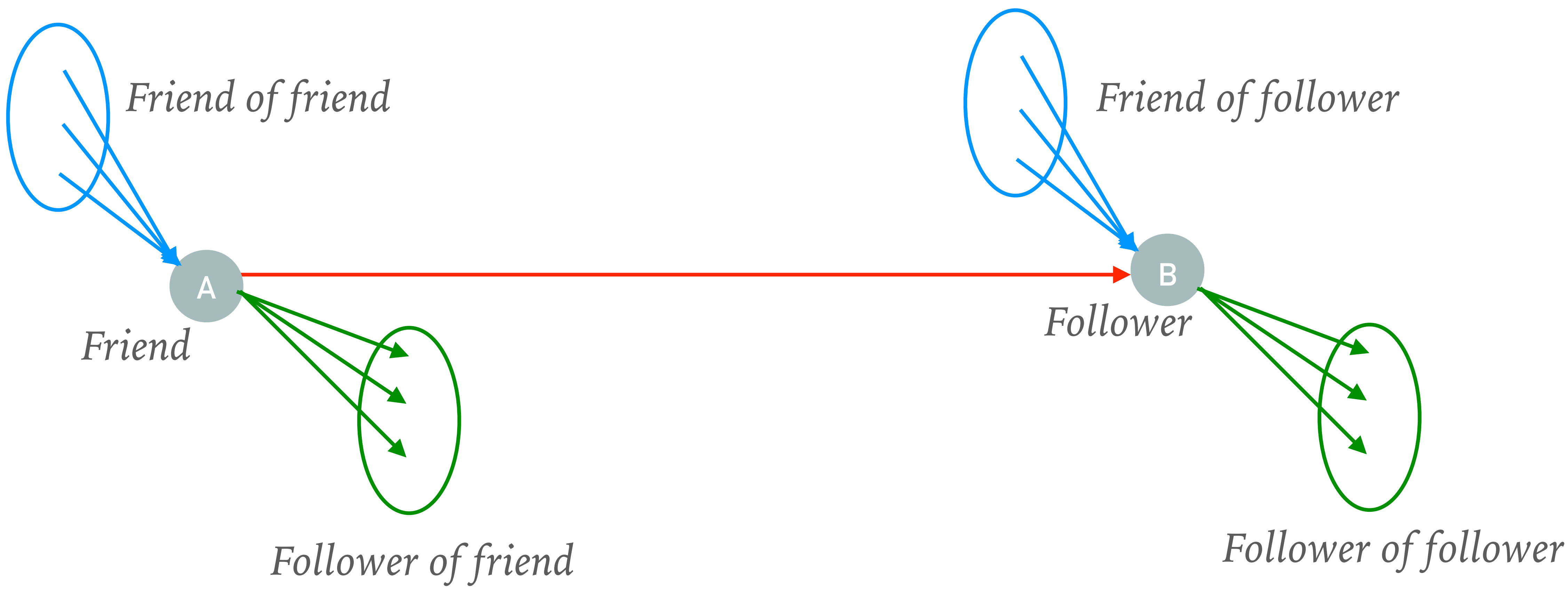
1. In what situations friendship paradox exists in directed networks?
2. How friendship paradox related to perception bias of individuals?
3. How we can get advantage from friendship paradox to estimate actual global prevalence?

NOTATION

- G = (V, E) is a **directed** network.
- Degree:
  - out-degree: number of **followers**
  - in-degree: number of **friends**
- Random variables:
  - X: random **node** $\mathbb{P}(X = v) = \frac{1}{N}$
  - Y: random **friend** $\mathbb{P}(Y = v) = \frac{d_o(v)}{\sum_{v' \in V} d_o(v')}$,
  - Z: random **follower** $\mathbb{P}(Z = v) = \frac{d_i(v)}{\sum_{v' \in V} d_i(v')}$,

FRIENDSHIP PARADOX IN DIRECTED NETWORKS

- **Friends** and **Followers**
- There are 4 types of paradox:



THEOREM 1

- In **all** directed networks:
  - Random **friend** Y has more **followers** than a random node X, on average:

$$\mathbb{E}\{d_o(Y)\} - \bar{d} = \frac{\mathrm{Var}\{d_o(X)\}}{\bar{d}} \geq 0.$$

  - Random **follower** Z has more **friends** than a random node X, on average:

$$\mathbb{E}\{d_i(Z)\} - \bar{d} = \frac{\mathrm{Var}\{d_i(X)\}}{\bar{d}} \geq 0.$$

  - d = average in-degree = average out-degree

THEOREM 2

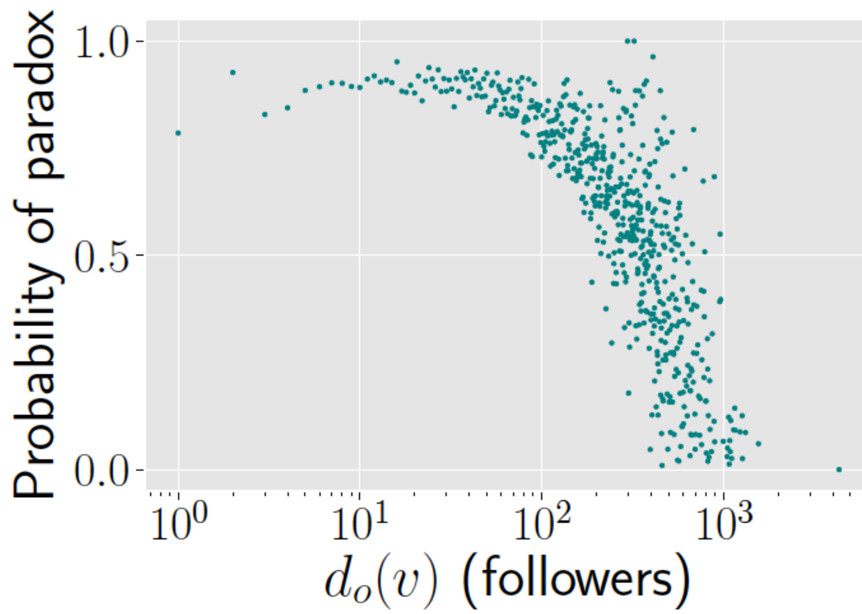
- If in-degree and out-degree of a random node X are **positively correlated**:
  - Random **friend** Y has more **friends** than a random node X, on average:

$$\mathbb{E}\{d_i(Y)\} - \bar{d} = \frac{\mathrm{Cov}\{d_i(X), d_o(X)\}}{\bar{d}} \geq 0.$$

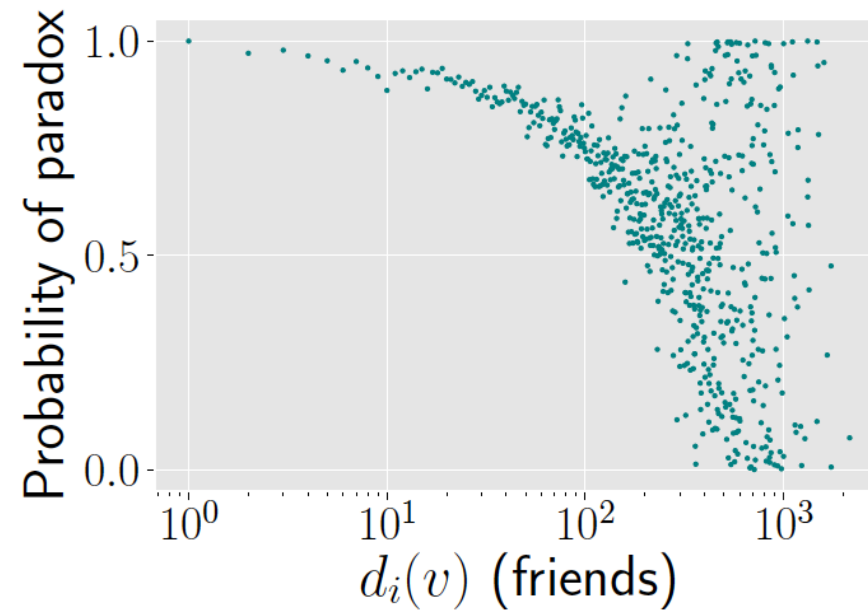
  - Random **follower** Z has more **followers** than a random node X, on average:

$$\mathbb{E}\{d_o(Z)\} - \bar{d} = \frac{\mathrm{Cov}\{d_i(X), d_o(X)\}}{\bar{d}} \geq 0.$$

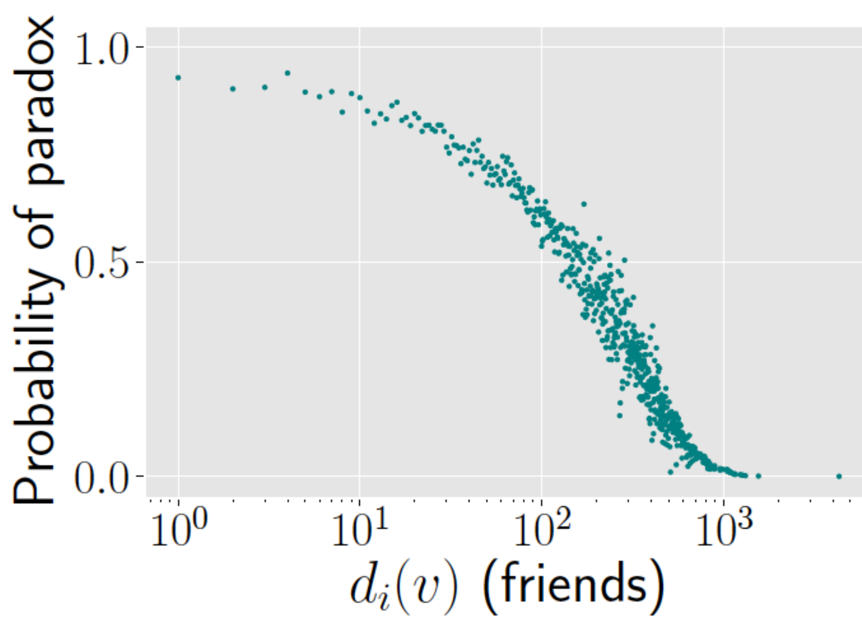
FRIENDSHIP PARADOX ON TWITTER NETWORK



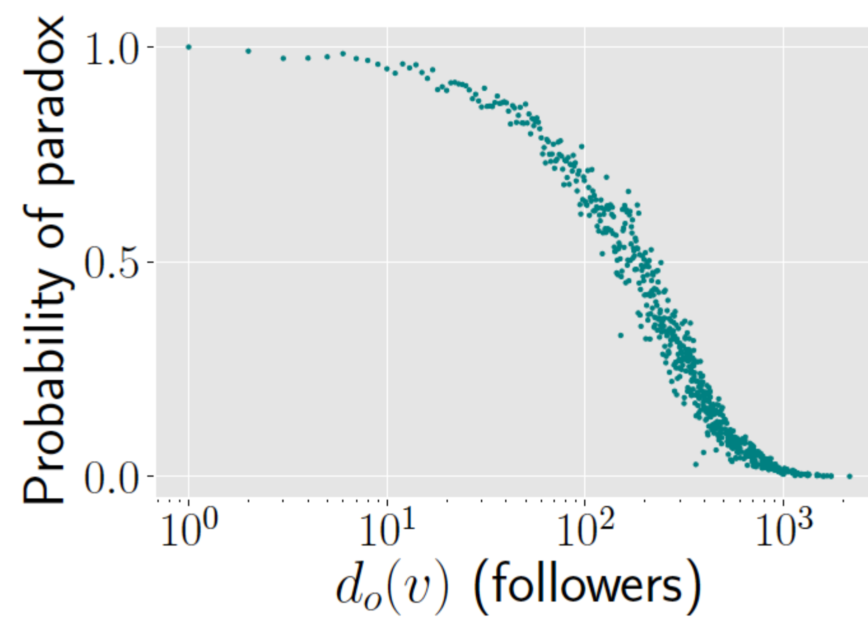
(a) Friends have more followers



(b) Followers have more friends



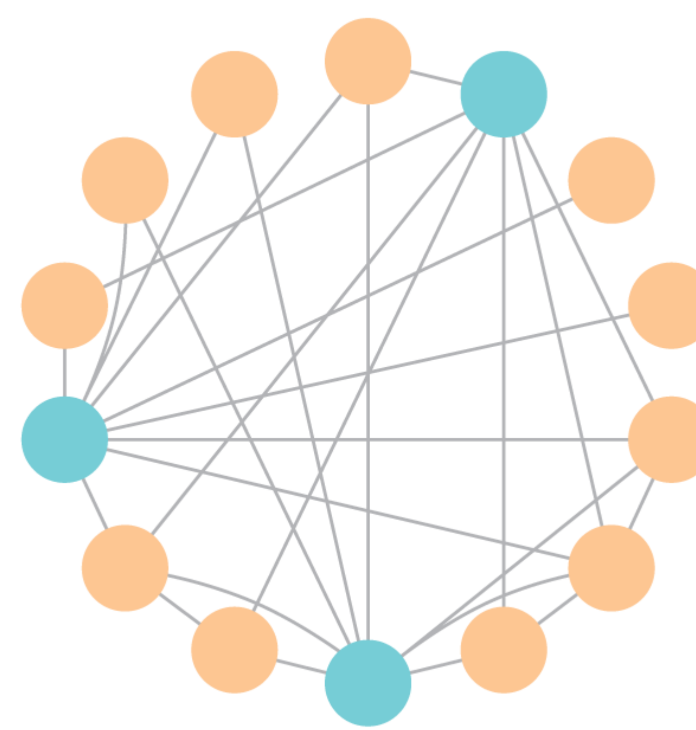
(c) Friends have more friends



(d) Followers have more followers

PERCEPTION BIAS

- When nodes have distinguishing traits, friendship paradox can bias perceptions of those traits.
- People look at their neighborhood to estimate the popularity of a topic.
- For example in twitter, the popularity of a hashtags: **#icebucketchallenge, #ferguson, #mikebrown, #sxsw**



ATTRIBUTE F

- f is a binary function $f : V \to \{0, 1\}$
- In twitter, for each hashtag we have a function
- $f(v) = 0$ means node v did not use hashtag.
- $f(v) = 1$ means node v used hashtag.
- We want to see in what situations a hashtag has perception bias.

GLOBAL PERCEPTION BIAS

- Global bias is defined as

$$B_{global} = \mathbb{E}\{f(Y)\} - \mathbb{E}\{f(X)\}$$

- Global Bias is difference between:
  - **global prevalence of attribute among friends** (expectation)
  - **actual global prevalence of attribute** (reality).

- **Theorem 3**:

$$\mathbb{E}\{f(Y)\} - \mathbb{E}\{f(X)\} = \frac{\mathrm{Cov}(f(X), d_o(X))}{\bar{d}} = \frac{\rho_{d_o,f}\sigma_{d_o}\sigma_f}{\bar{d}}$$

- Larger the covariance of out-degree and attribute f, larger the global bias.

LOCAL PERCEPTION BIAS

- Define $q_f(v)$ as fraction of friends with attribute:

$$q_f(v) = \frac{\sum_{u \in Fr(v)} f(u)}{d_i(v)}$$

- Define local bias:

$$B_{local} = \mathbb{E}\{q_f(X)\} - \mathbb{E}\{f(X)\}$$

- Local Bias is difference between:
  - **expected fraction of friends with attribute** (expectation)
  - **actual global prevalence of attribute** (reality).

THEOREM 4

- Local bias is positive if

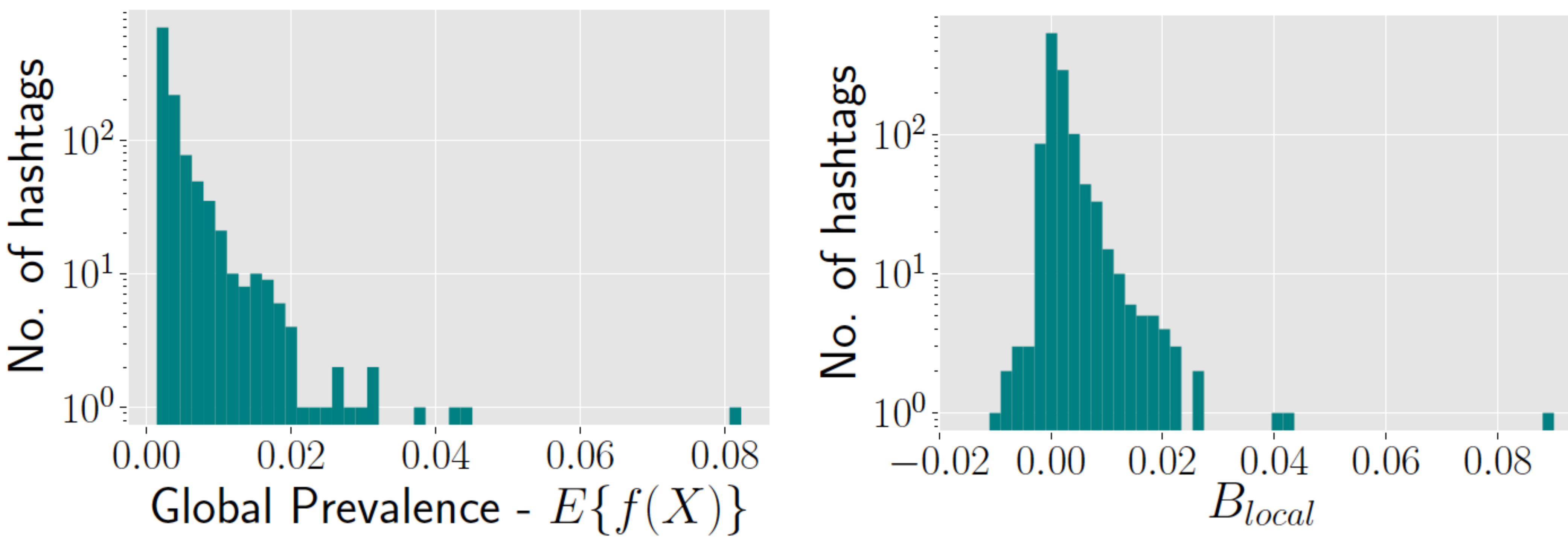
$$\mathrm{Cov}\{f(X), d_o(X)\} \geq 0 \quad \textit{and},$$
$$\mathrm{Cov}\{f(U), \mathcal{A}(V) | (U, V) \sim \mathrm{Uniform}(E)\} \geq 0.$$

- where

$$\mathcal{A}(v) = \frac{1}{d_i(v)}.$$

- Local bias is positive if:
  - Higher degree nodes (nodes with **high influence**) tend to have the attribute.
  - Lower degree nodes(nodes with **high attention per friend**) tend to follow nodes with attribute.

CHARACTERISTICS OF HASHTAGS



(a) **(b)**

- The figure shows the histogram of the prevalence of the 1,153 most popular hashtags.
- 865 hashtags having positive bias, meaning that they appear more popular than they really are.

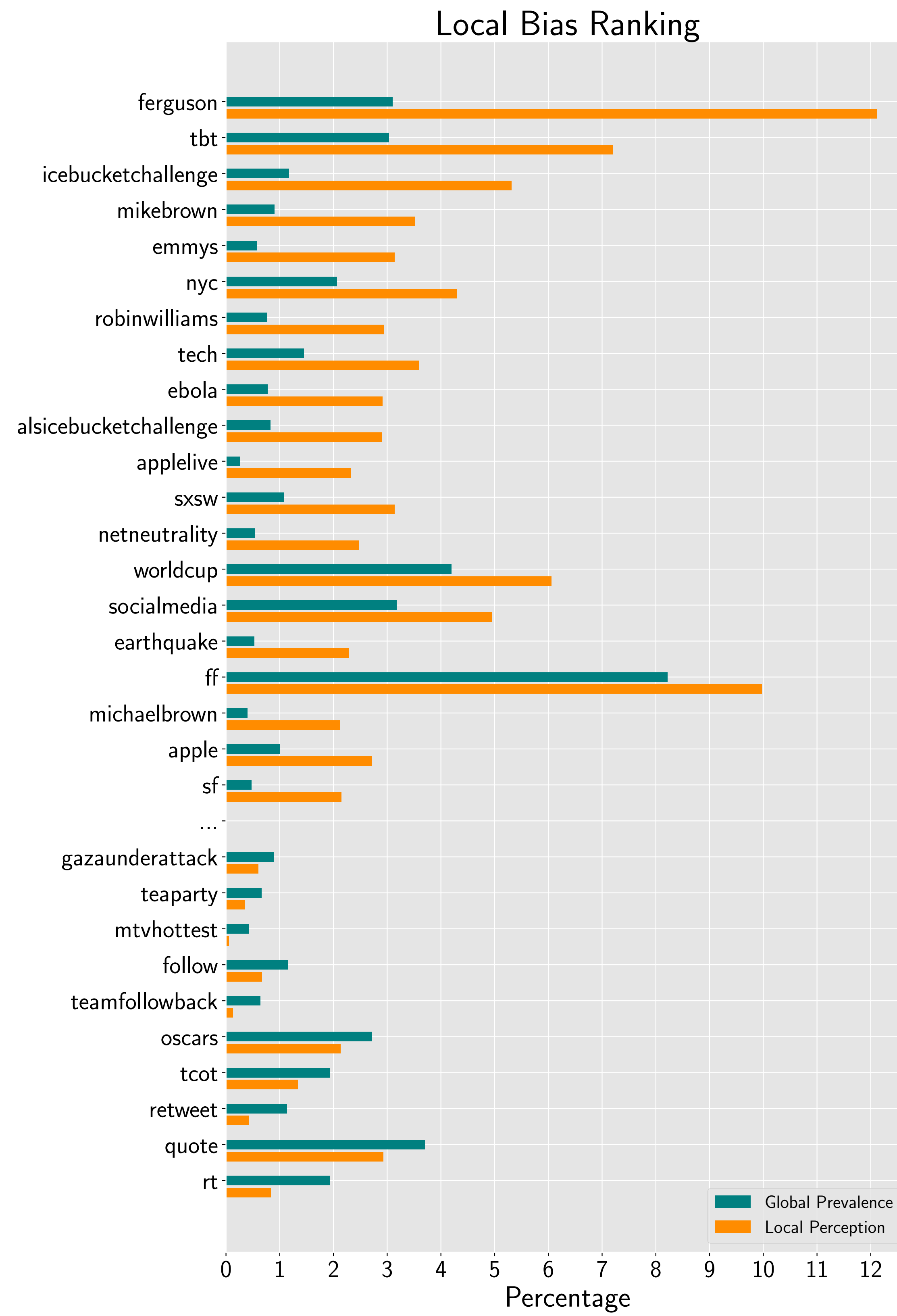
RANKING BASED ON LOCAL BIAS

Most positive biased Hashtags:

- **Social movements** (#ferguson, #mikebrown, #michaelbrown)
- **Memes and current events** (#icebucketchallenge, #ebola, #netneutrality)
- **Sport and entertainment** (#emmys, #sxsw, #robinwilliams, #applelive, #worldcup)

Most negative biased Hashtags:

- **getting more followers** (#tfb, #followback, #follow, #teamfollowback)
- **more retweets** (#shoutout, #pjnet, #retweet, #rt).
- #oscars, #tcot and #rt are globally prevalent but their local bias is negative.



INDIVIDUAL-LEVEL PERCEPTION BIASES

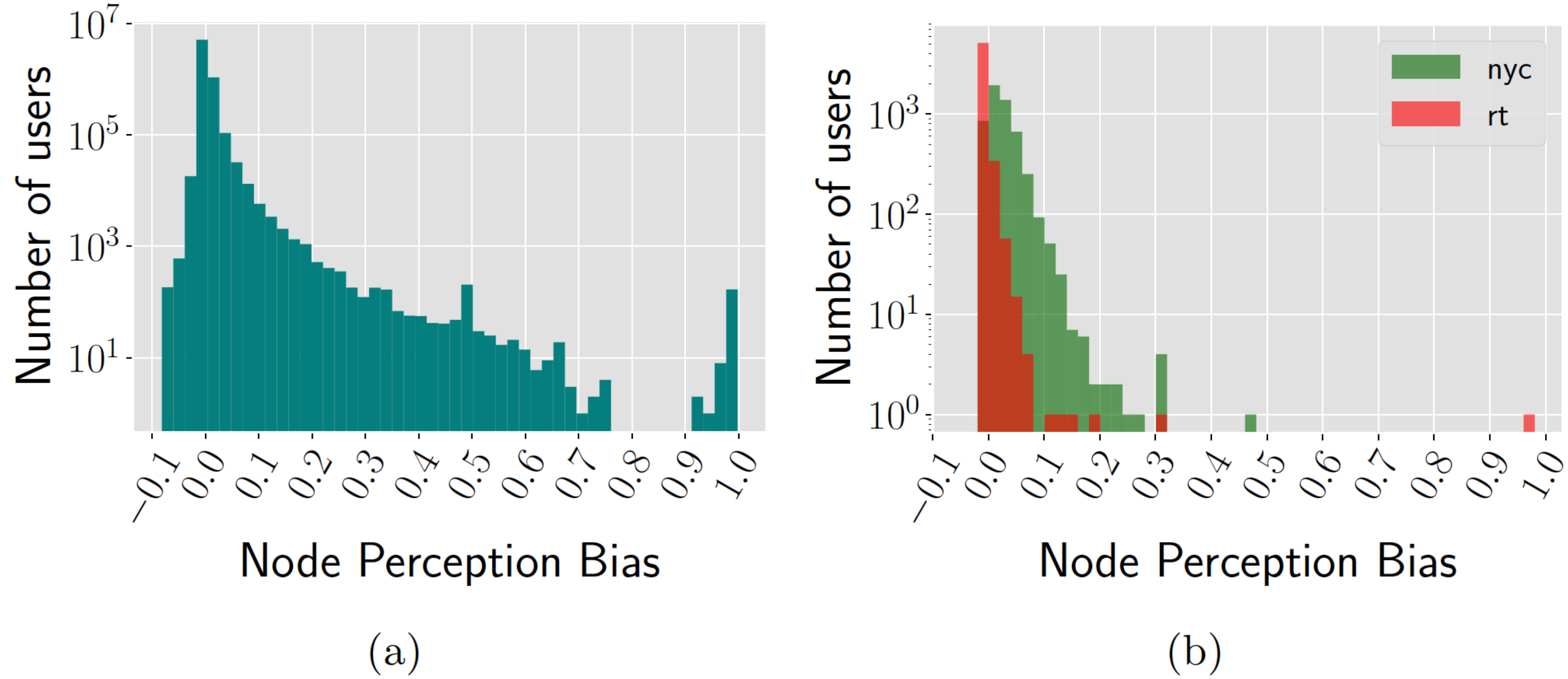


Figure 4: Individual-level perception bias $q_{f_h}(v) - \mathbb{E}\{f(X)\}$ for (a) all hashtags h and all nodes $v \in V$, and (b) for two hashtags with similar global prevalence, but with positive (*#nyc*) and negative (*#rt*) B_{local}. This illustrates that most hashtags are positively biased for individuals, with bias levels that do not depend on global prevalence.

POLLING

- How to estimate the actual global prevalence of an attribute in the presence of such perception bias?
- With **limited budget:** poll at most **b** individuals.
- For example: How to estimate fraction of democrats / republicans in a network?

PREVIOUS WORKS

- **The accuracy of a poll depends on two key factors:**
  - The method of sampling individuals.
  - The question presented to them

- **Polling**:
  1. **Intent (IP)**: [**b** random nodes] Who will you vote for?
  2. **Expectation**: [**b** random nodes] Who do you think will win?
  3. **Node Perception (NPP)**: [**b** random nodes] What fraction of your friends vote for X?

- Mean square error

$$\mathrm{MSE}\{T\} = \mathbb{E}\{(T - \mathbb{E}(f(X))^2\} = \mathrm{Bias}\{T\}^2 + \mathrm{Var}\{T\}$$

FOLLOWER PERCEPTION POLLING (FPP)

- Based on Theorem 1, random follower Z has more friends than a random node X. So, the **variance** of estimate would be smaller.
- [**b** random **followers**] What fraction of your friends vote for X?

Algorithm 1: Follower Perception Polling (FPP) Algorithm

Input: Graph $G = (V, E)$, perceptions $q_f : V \to \mathbb{R}^+$, sampling budget b.

Output: Estimate $\hat{\mathrm{f}}_{\mathrm{FPP}}$ of $\mathbb{E}\{f(X)\} = \frac{\sum_{v \in V} f(v)}{N}$.

(1) Sample a set $S \subset V$ of b followers independently from the distribution

$$p_v = \frac{d_i(v)}{\sum_{v' \in V} d_i(v')}, \quad \forall v \in V.$$

(2) Compute the estimate

$$\hat{\mathrm{f}}_{\mathrm{FPP}} = \frac{1}{b} \sum_{v \in S} q_f(v). \tag{17}$$

BIAS OF FPP

- Mean square error of Polling

$$\mathrm{MSE}\{T\} = \mathbb{E}\{(T - \mathbb{E}(f(X))^2\} = \mathrm{Bias}\{T\}^2 + \mathrm{Var}\{T\}$$

- Bias of the estimate (error) for FPP algorithm is Global Bias:

$$\begin{aligned}\mathrm{Bias}(\hat{\mathrm{f}}_{\mathrm{FPP}}) &= \mathbb{E}\{\hat{\mathrm{f}}_{\mathrm{FPP}}\} - \mathbb{E}\{f(X)\}\\ &= B_{global}\end{aligned}$$

BOUND ON VARIANCE OF FPP

- The variance of FPP algorithm is bounded by

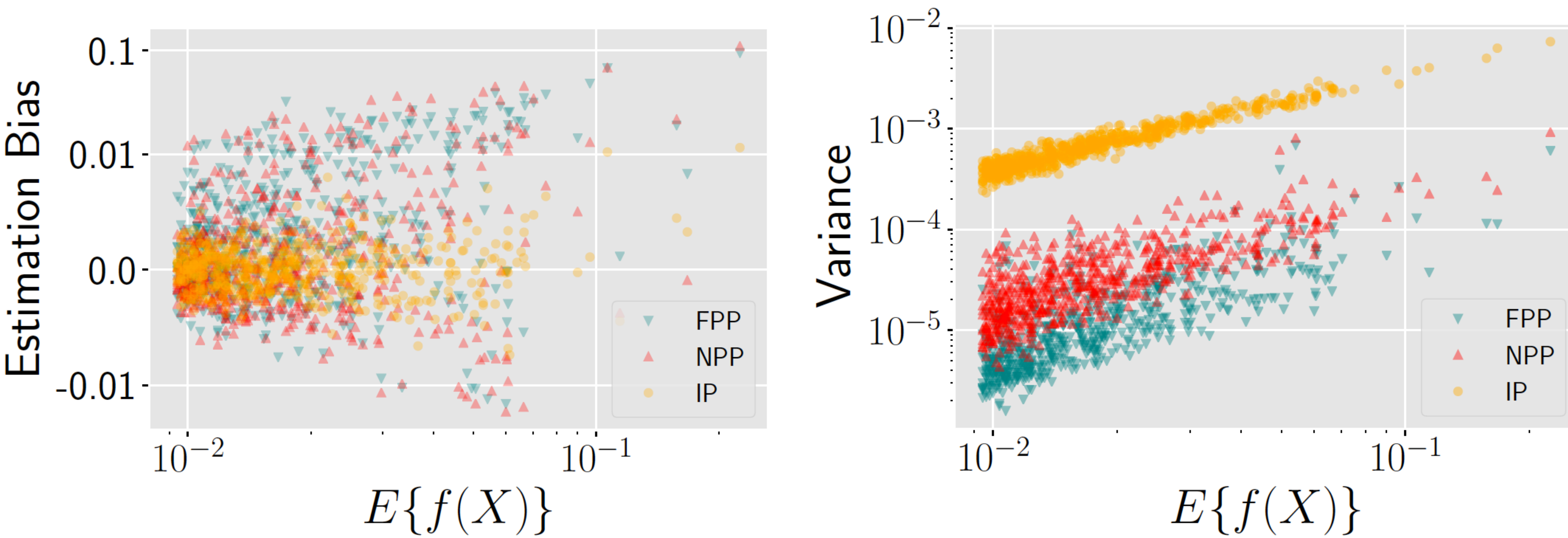
$$\mathrm{Var}(\hat{\mathrm{f}}_{\mathrm{FPP}}) \le \frac{1}{bM} \lambda_2 ||D_o^{1/2} f||^2$$

  - b is budget
  - M is number of edges
  - λ_2 is the second largest eigenvalue of Bibliographic coupling matrix.

- Smaller variance with:
  - Higher budget b
  - Lower correlation of out-degree and attribute
  - Good expansion (smaller λ_2) and less bottleneck.

EMPIRICAL RESULTS

➤ Sample budget: b = 25 (0.5% of the network size)



Intent Polling - IP : asks random users whether they used a hashtag

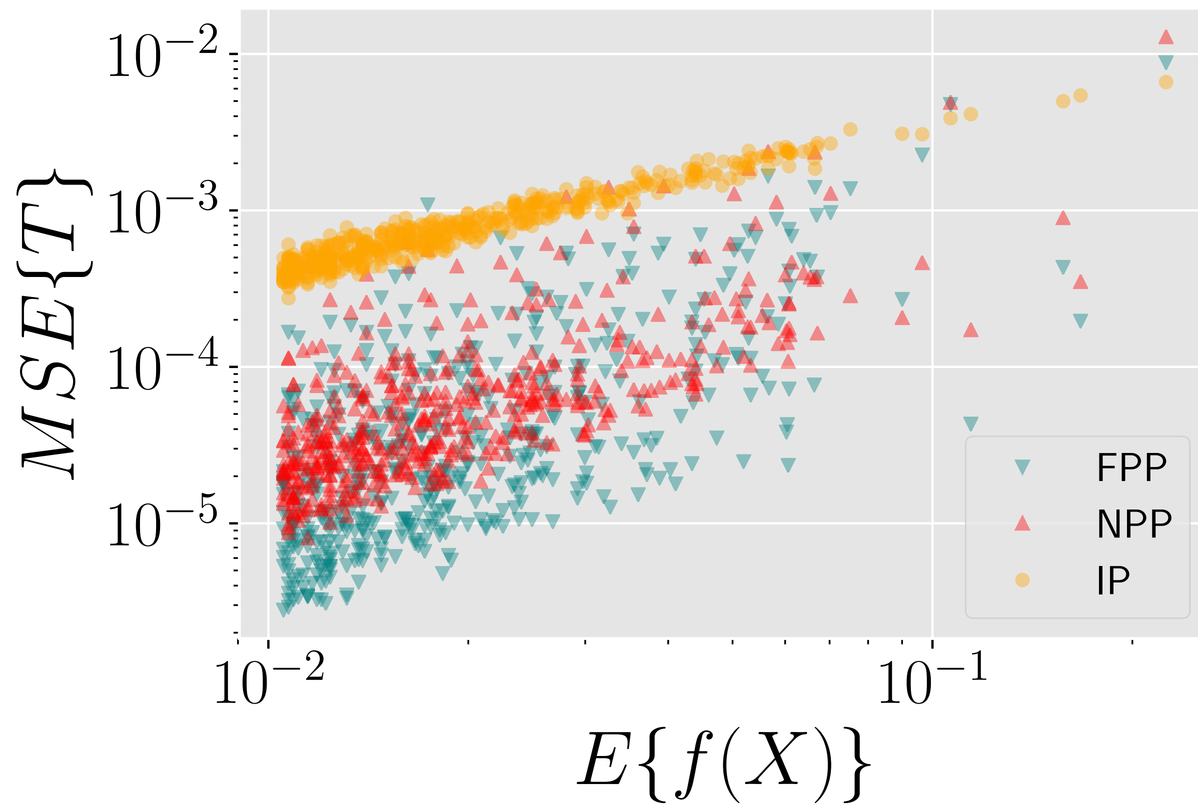
Node Perception Polling - NPP : asks random users what fraction of their friends used the hashtag.

Follower Perception Polling - FPP : asks random followers what fraction of their friends used the hashtag.

MEAN SQUARED ERROR (MSE)

- Accuracy of algorithms in terms of both bias and variance:

$$\mathrm{MSE}\{T\} = \mathbb{E}\{(T - \mathbb{E}(f(X))^2\} = \mathrm{Bias}\{T\}^2 + \mathrm{Var}\{T\}$$



- For b=25 (0.5% of the network size):
  - For 99% of hashtags **FPP** out-performs IP
  - For 81% of hashtags **FPP** out-performs NPP

SUMMARY

- We identify conditions under which friendship paradox can distort how popular some attribute is perceived.
- We validated these findings empirically using data from the Twitter social network.
- Identified hashtags that appeared several times more popular than they actually were, due to local perception bias.
- Presented an algorithm that leverages friendship paradox in directed networks to efficiently (in a MSE sense) estimate the true prevalence of an attribute.

OPEN QUESTIONS

- Perception bias may help amplify the spread of such influence by making them appear more common.
- How do perception biases and diffusion dynamics in networks relate?

QUESTIONS?