Weg2Vec: Event Embedding for Temporal Networks

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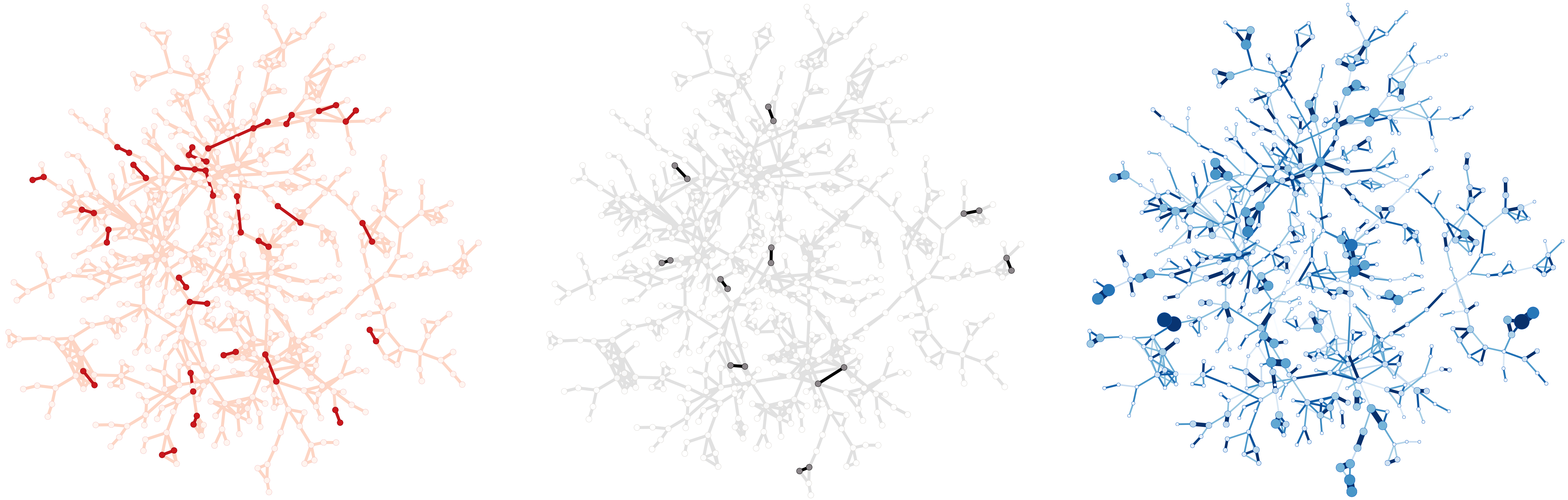
Temporal Networks

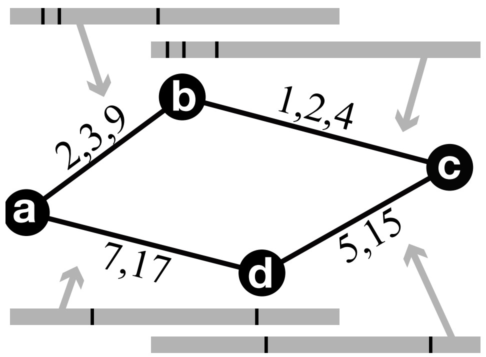


- Interactions between entities are not present always but varying in time (Holme, Saramaki 2012)
    - *Calls, SMS, f2f, @mentions, collaborations, transportation networks…*

Representation of temporal networks

1. Temporal Graph: $G_t = (V, E, T_e)$

- V: set of vertices
- E: set of edges
- $T_e=\{t_1, t_2, \ldots, t_n\}$: set of times when edge e is active



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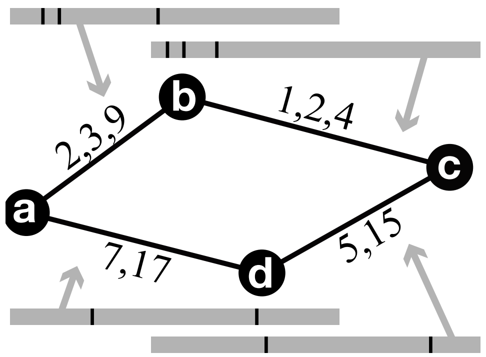

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2. Contact sequence

(Similar representation is called *link streams, (Latapy et al. 2018)*

$$E \subset T \times V \times V (\times \prod_i A_i^e \times L)$$

where

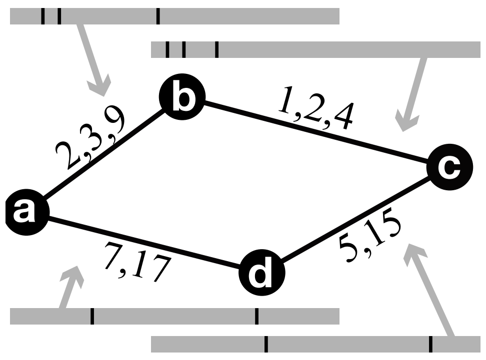
- *T* is the set of time stamps
- *V* is the set of interacting entities
- A^e are event attribute set e.g. duration, cost, etc,
- L is a location set

- sequence of events $ev \in E$

$$ev(t, u_{beg}, v_{end}, a_1, a_2, , \ldots, loc_{beg}, loc_{end})$$

t_1	a	b
t_2	a	c
t_4	e	f
t_{10}	f	a
…		

Representation of temporal networks



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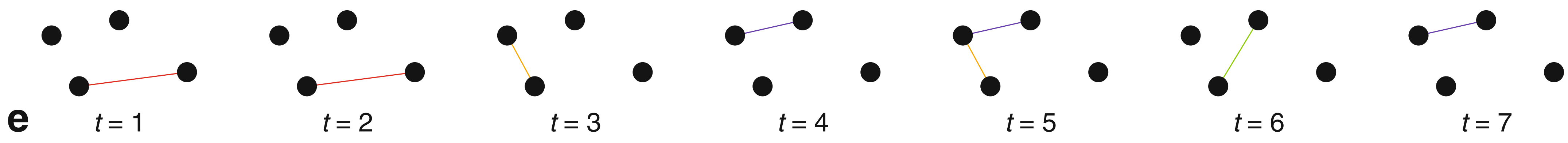
	…	

3. Graphlet or snapshot representation

- Set of graphs representing aggregated interactions happening at the same time or interval
- Can be represented as a dynamic adjacency matrix $A_{ij}(t)$
- Can be represented as a multiplex network



Representation of temporal networks

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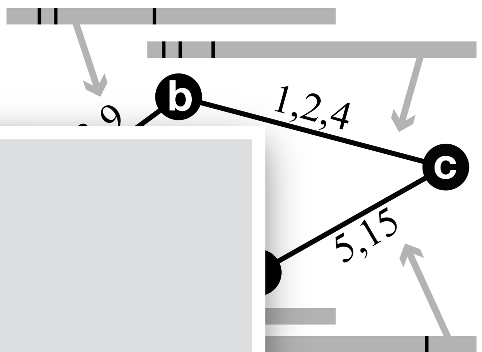
- V: set of vertices



2. Con

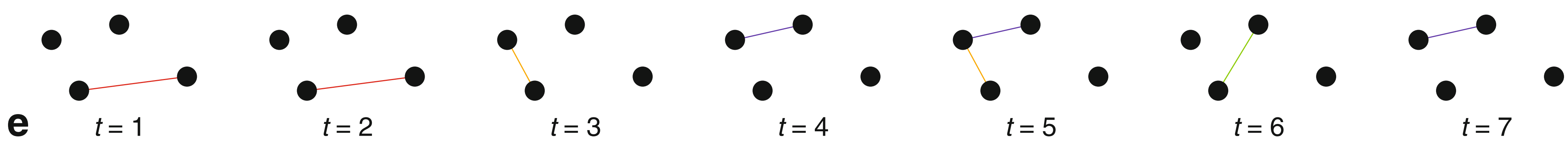
(Similar re

a	b
a	c
e	f
f	a
…	

3. Gra

Computational difficulties:

- Expensive to measure **temporal centralities and similarities**
  - any node/link character vary in time
- Expensive to compute **time respecting paths**
  - depends on time and seed
- Expensive to detect **causal correlations**
  - interactions are not independent but form local correlated patterns
- Expensive to simulate **dynamical/epidemic processes**
  - They must be seeded from every time point and every node



Temporal
Event
Graphs

Time-respecting paths

- Temporal equivalent of topological path in static graphs
- Consider a temporal contact network (for simplicity without durations)
- Any path between node has to respect the timing and ordering of events!

Definition

- Time-respecting path between node *a* and *b* is a set of events

$$\{(a, v, t_1), (v, w, t_2), \ldots, (y, b, t_n)\}$$

such that $t_1 < t_2 < \ldots < t_n$ and consecutive events are adjacent

(i.e. time ordered and share at least one node)

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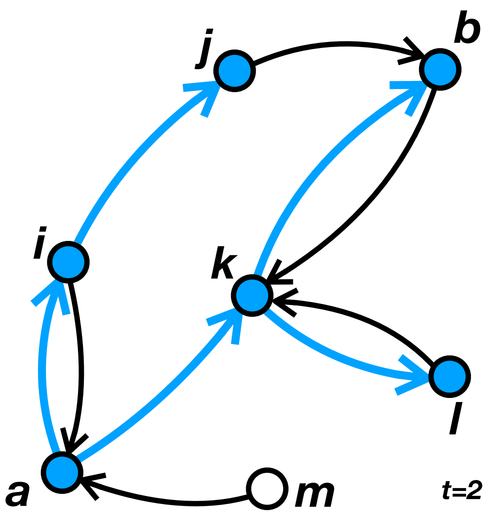
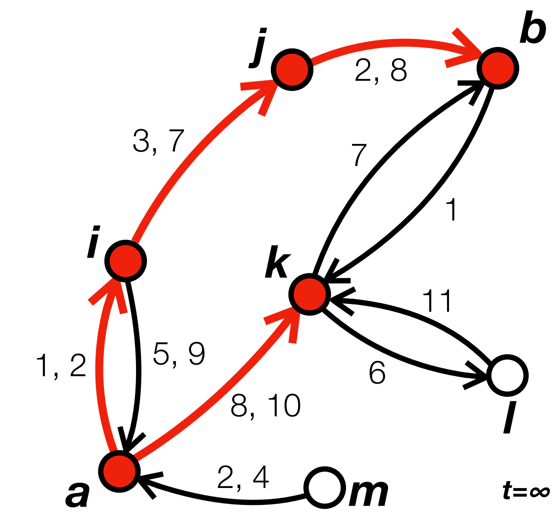


static path



temporal path

Properties

- Reachable set of nodes are limited
- No reciprocity: the existence of the path a-b does not guarantee the existence of a b-a path
- No transitivity: the existence of the path a-b and b-c does not guarantee that there is a a-b-c path
- Time dependency: paths begin and end at certain times; if there is a path a-b that begins at t, this doesn't guarantee a path at $t' > t$
- **They determine the spread of information thus the outcome of any collective phenomena**

Weighted event graphs

Temporal networks

$$G = (V, E, T) \quad \text{with events} \quad E \subset V \times V \times [0, T]$$

Adjacent events

Events $e = (a, b, t)$, $e' = (b, c, t')$ are adjacent $e \to e'$ if

- Share at least one common node
- $t < t'$

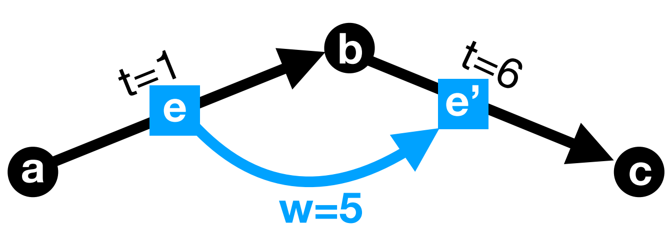
Weighted event graphs

Kivelä, Cambe, Saramäki, Karsai, *Sci. Rep. (2018)*
Mellor *J. Complex Netw. (2017).*

$D = (E, E_D, w)$ where

- nodes are events in G
- links $e_D \in E_D$ are adjacent events $e_D = e \to e'$
- weights are $w(e_D) = t' - t$
- δt threshold for weights: keeping adjacent events which are closer in time than δt

Temporal network



Weighted even graph

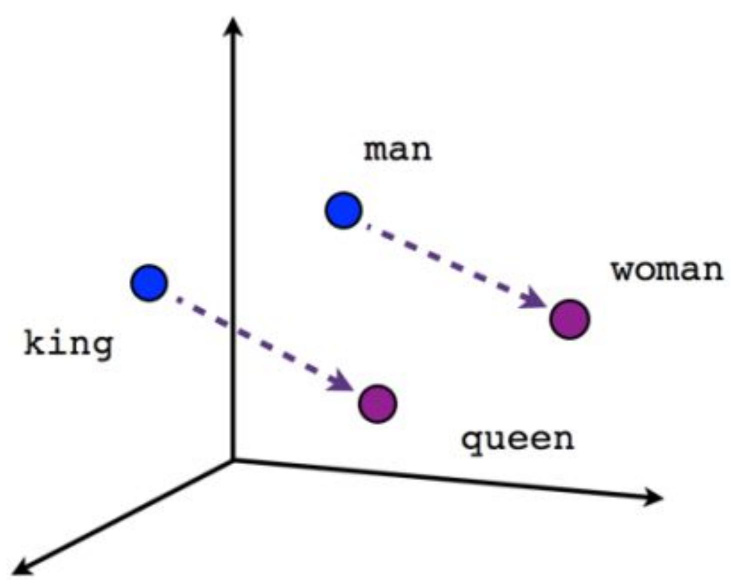
- **Static and lossless representation** of all temporal and structural information
- It is a weighted **directed acyclic graph (DAG)**
- **Superposition of every** (δt-connected) **time-respecting paths**
- Its connectedness determines the outcome of any dynamical process

Weg2Vec:
Event Embedding
for
Temporal Networks

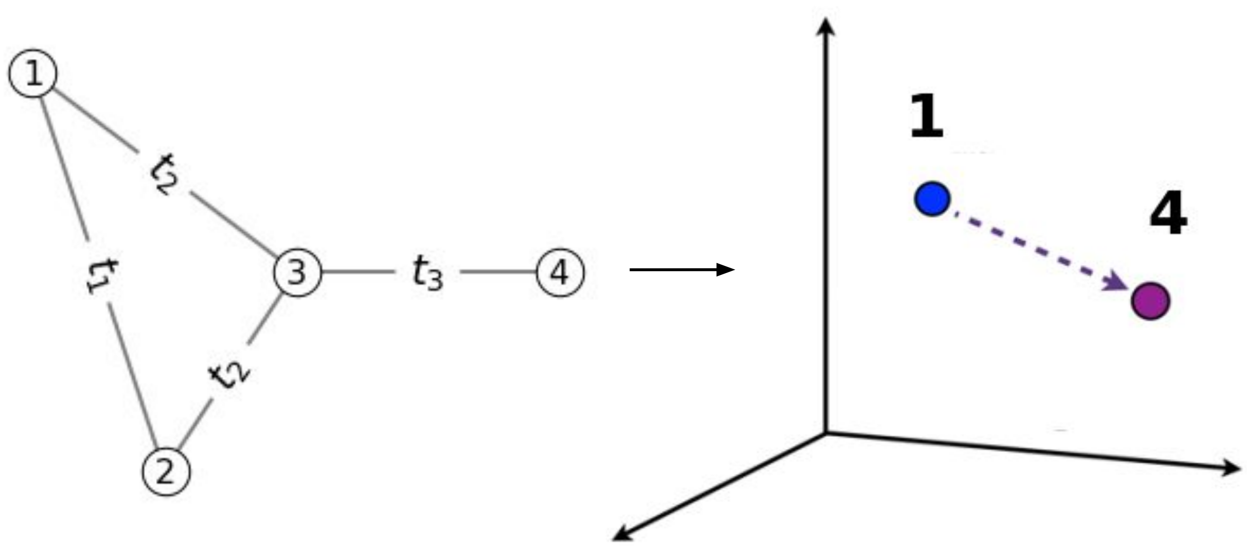
Temporal network embedding

- Learn low-dimensional representations
- Capture temporal and structural regularities in the network
- Various applications: node classification, link prediction...

… or the prediction of spreading outcome



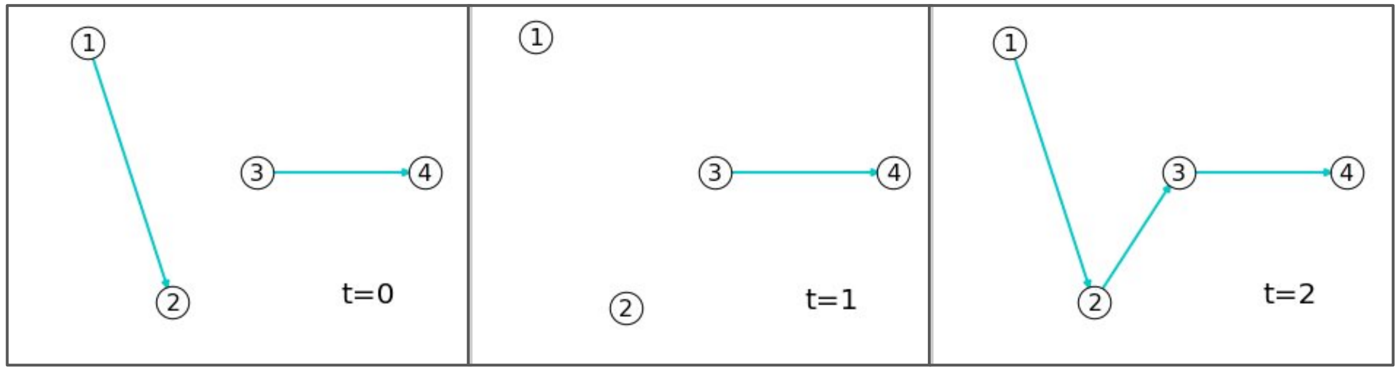
Mikolov, T., Chen, K., Corrado, G., & Dean, J. (2013). Efficient estimation of word representations in vector space.



Weg2Vec pipeline

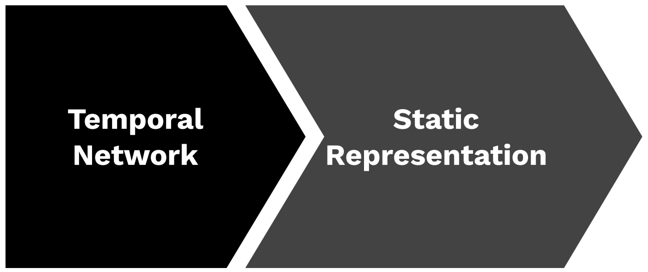
Temporal Network

TEMPORAL NETWORK

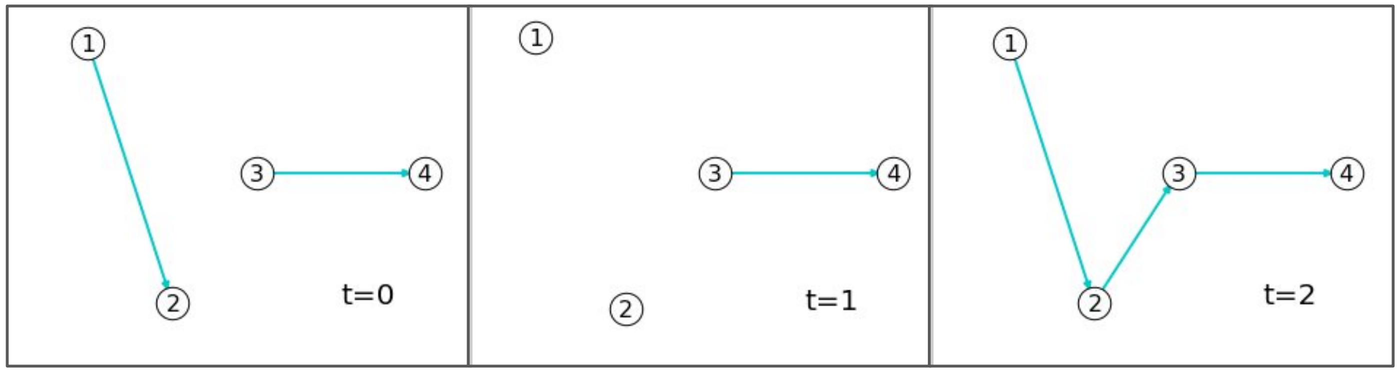


Network	Nodes	Events	Temporal Interval
Conference	113	10457	$\sim$ 2.5 days
Hospital	75	13650	$\sim$ 4 days
High School	327	36015	$\sim$ 2 days $*$
Primary School	236	35921	$\sim$ 8 hours $*$

Weg2Vec pipeline

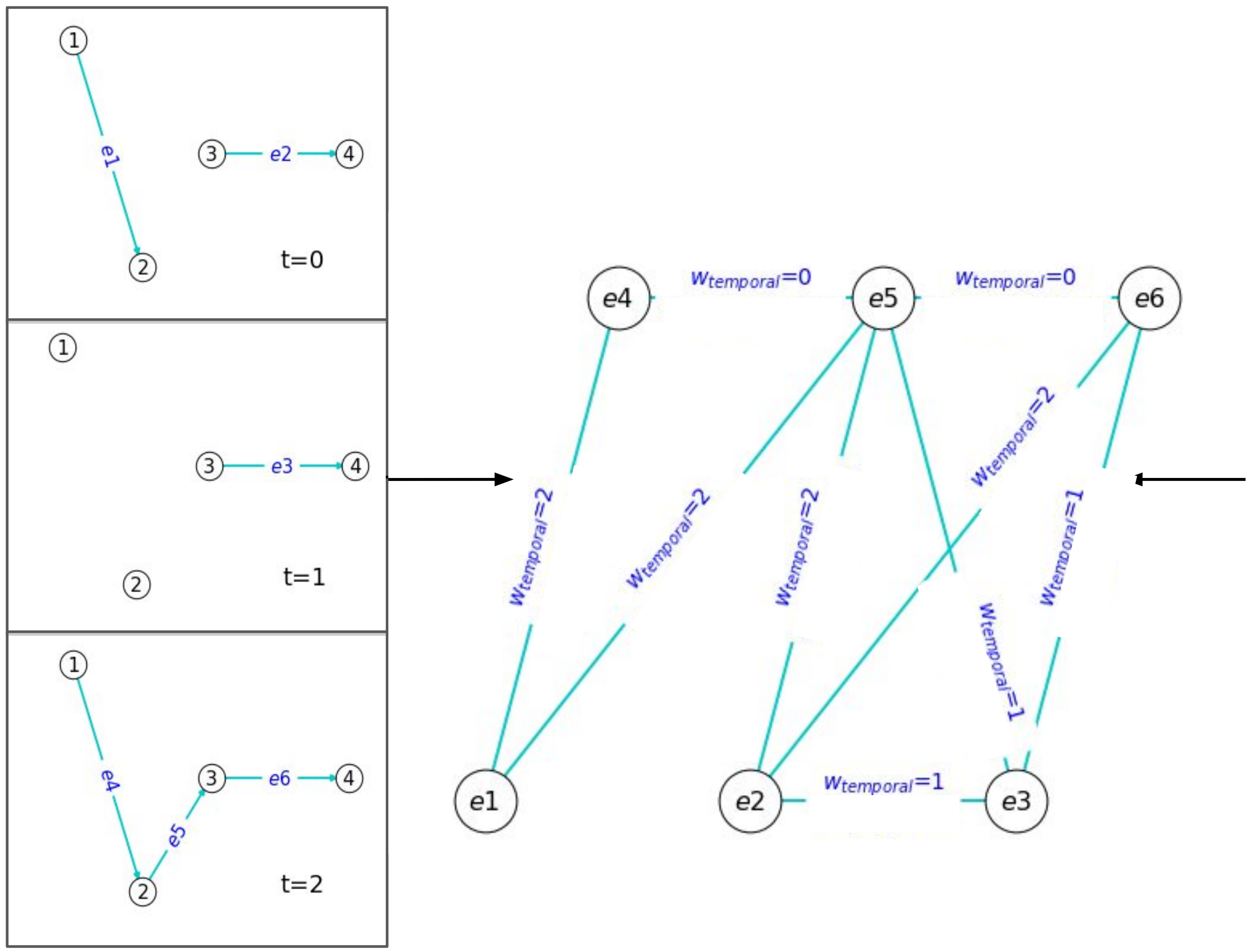


TEMPORAL NETWORK



Event embedding

WEIGHTED EVENT GRAPH

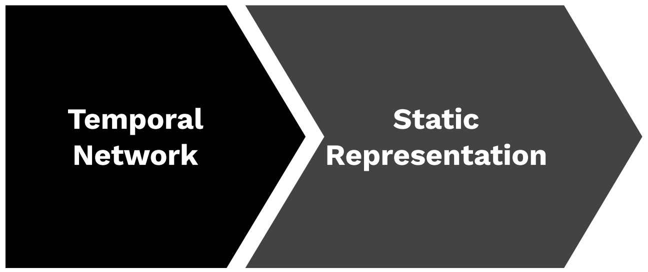


Event graph representation

- **Path (temporal) weight:**

$$w_{path}(e_k, e_l) = \frac{1}{1 + |t_k - t_l|}$$

Weg2Vec pipeline



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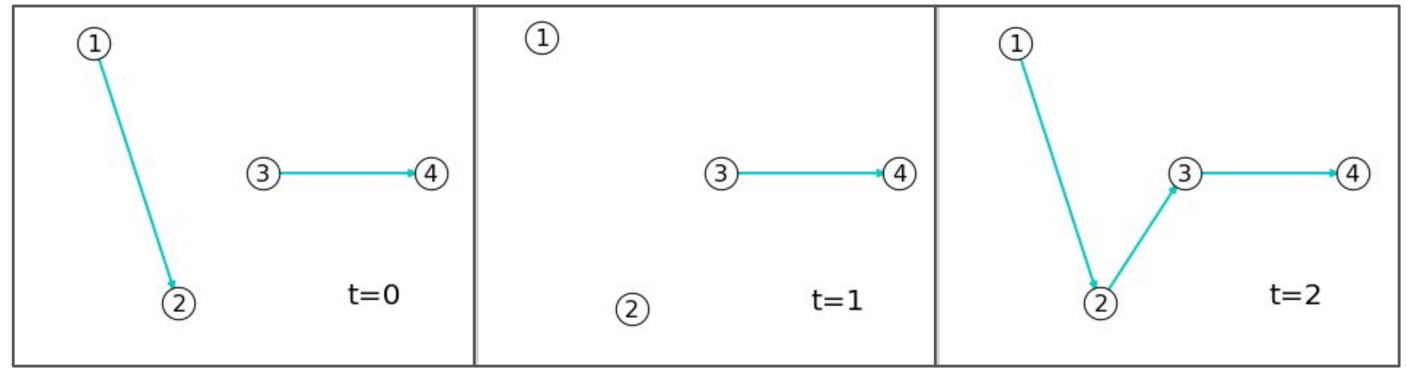


Event embedding

Event graph representation

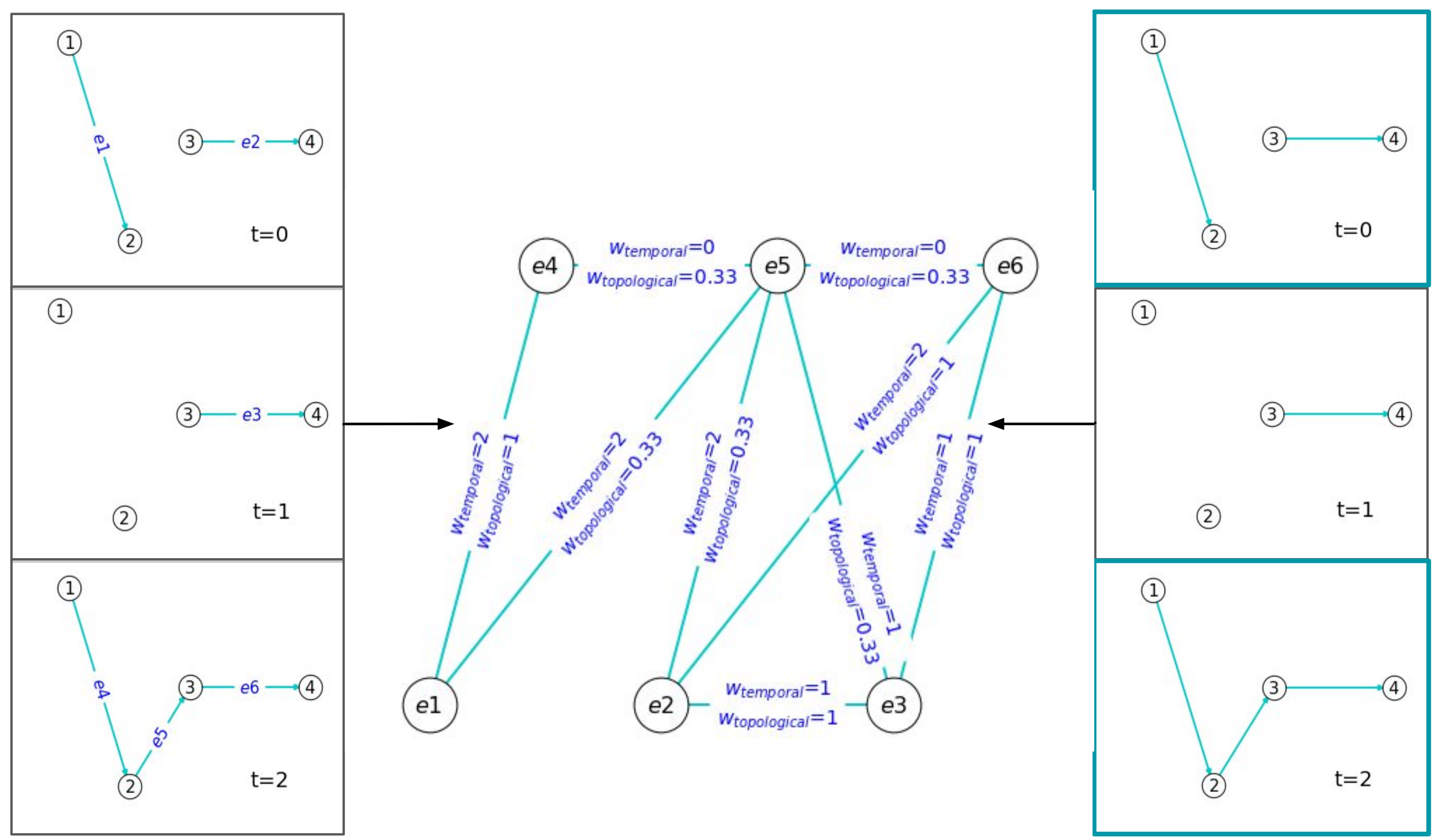
- **Path (temporal) weight:**

$$w_{path}(e_k, e_l) = \frac{1}{1 + |t_k - t_l|}$$

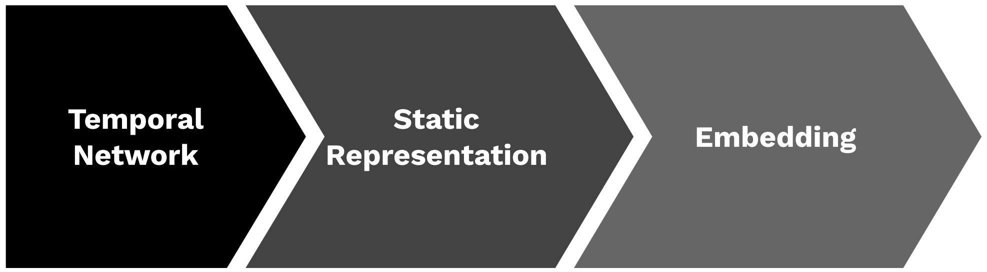
- **Co-occurance (topological) weight:** $w_{co-occ}(e_k, e_l)$
  - Number of co-occurance of δt adjacent events on the same pair of adjacent static links

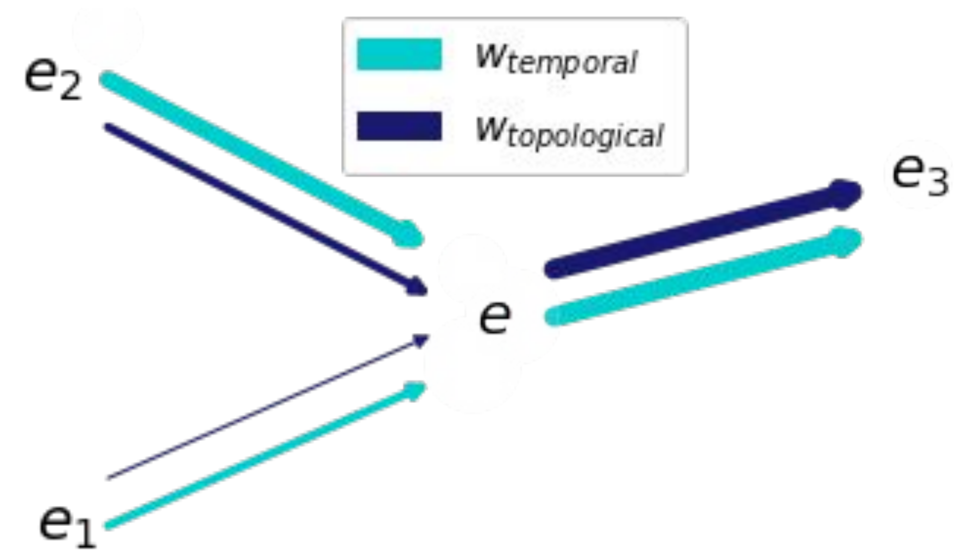
WEIGHTED EVENT GRAPH

LINK CO-OCCURENCES



Weg2Vec pipeline





Event embedding

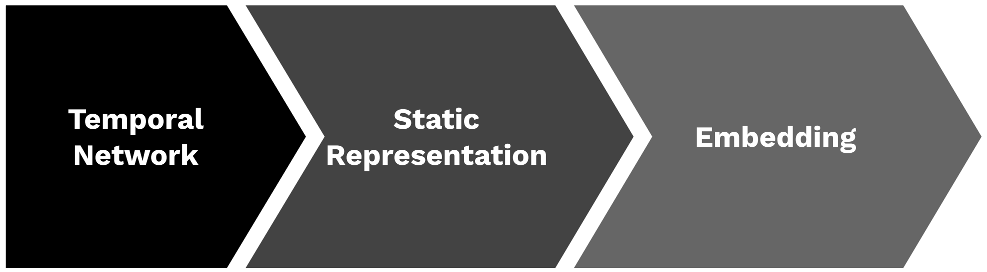
We rely on the **static representations of the temporal networks** to generate the **contexts** to be passed as input to **Word2Vec**

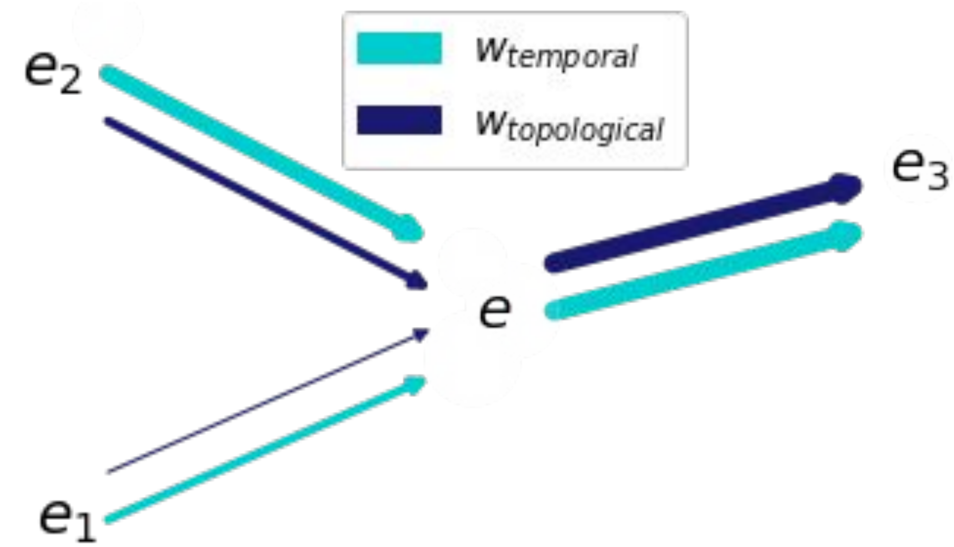
$$p(e_l) = \alpha \mathcal{F}(w_{path}(e_k, e_l) + (1-\alpha)\mathcal{F}(w_{co-occ}(e_k, e_l))$$

- Sample nb local **environments** of s length for each event by randomly choosing neighbours in the event graph with probability $p(e_l)$
- Sampling equally from the set of **past** (predecessors) and **future** (successor) adjacent events

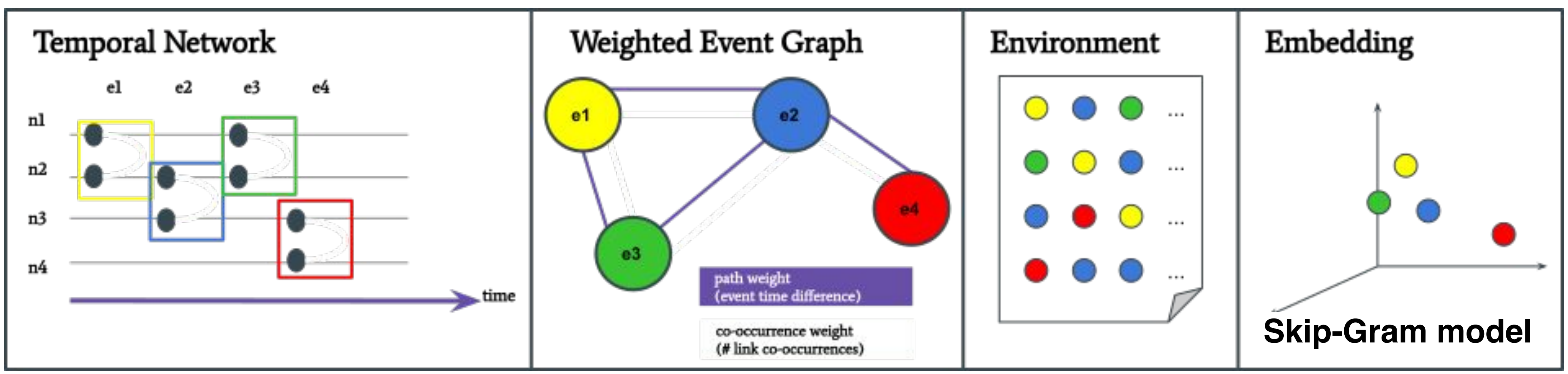
Identifies similarity between different events/nodes, which may be active at different times, but influence a similar set of nodes in the future

Weg2Vec pipeline





Event embedding



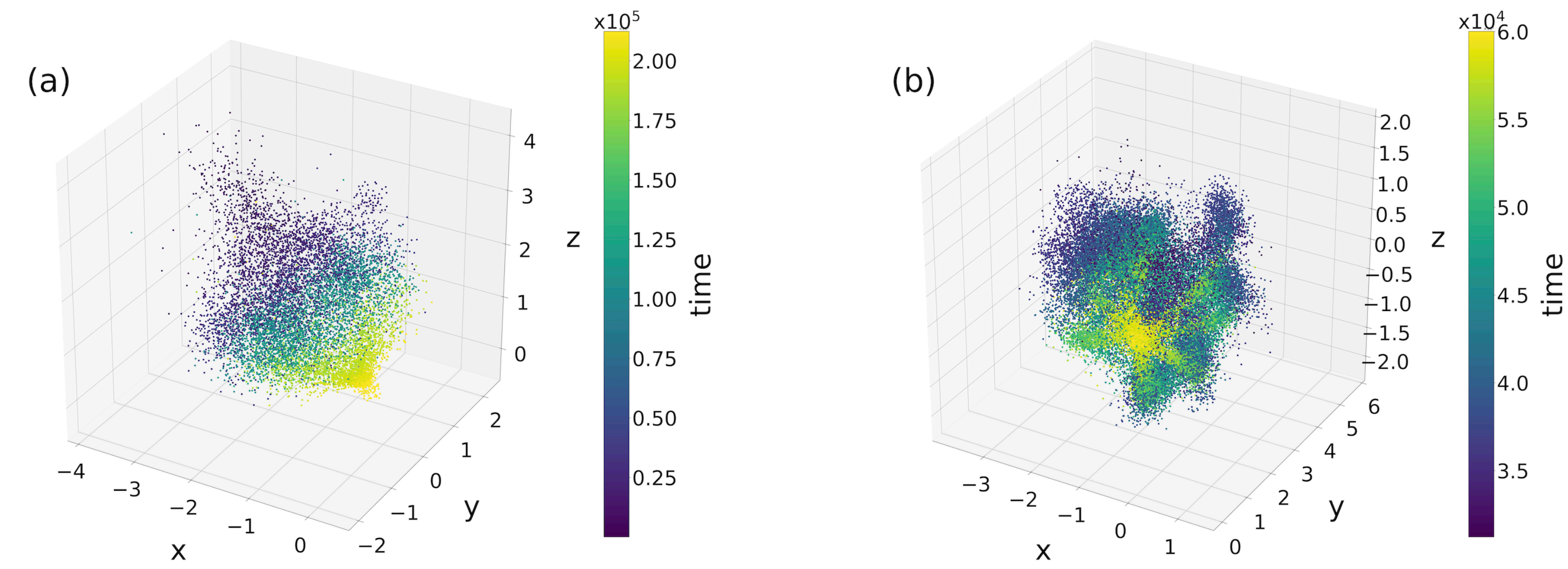
Parameters

- α - **balance parameter between temporal and topological contribution**
- d - **number of embedding dimensions**
- s and nb **- context parameters for environment sampling**

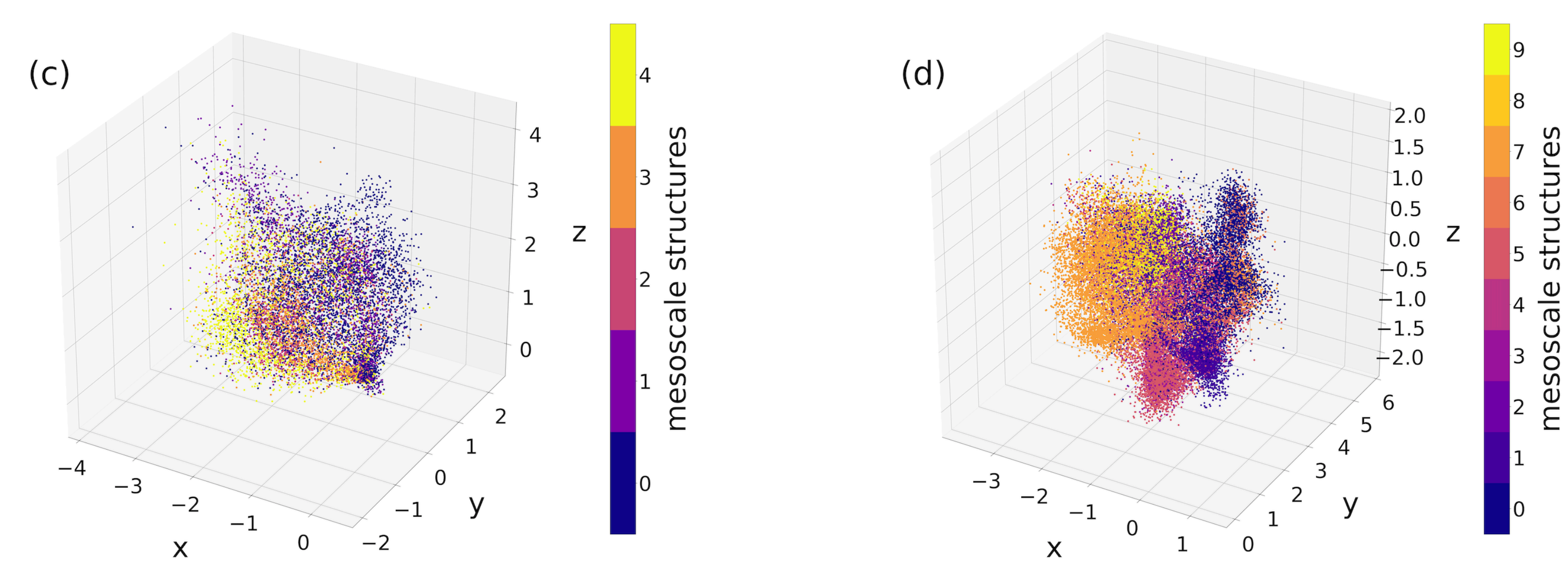
Weg2Vec - embedding

Embedding time & structure

Temporal ordering



Membership in mesoscale structures

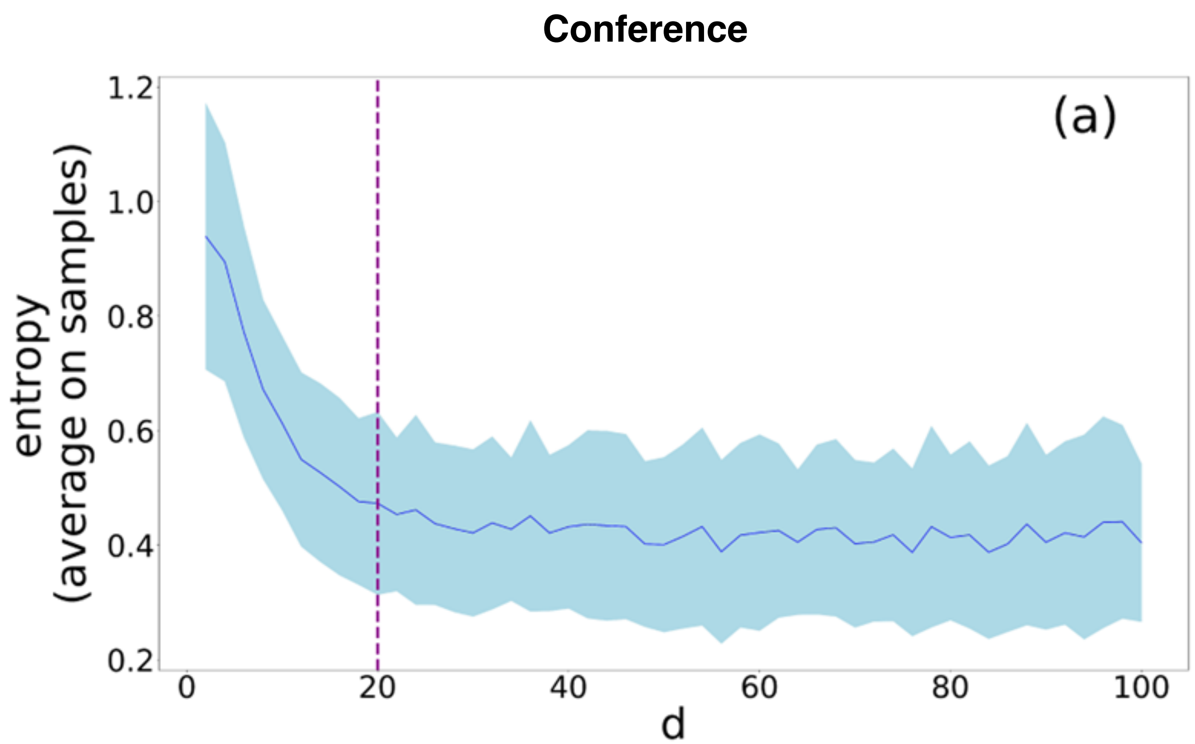


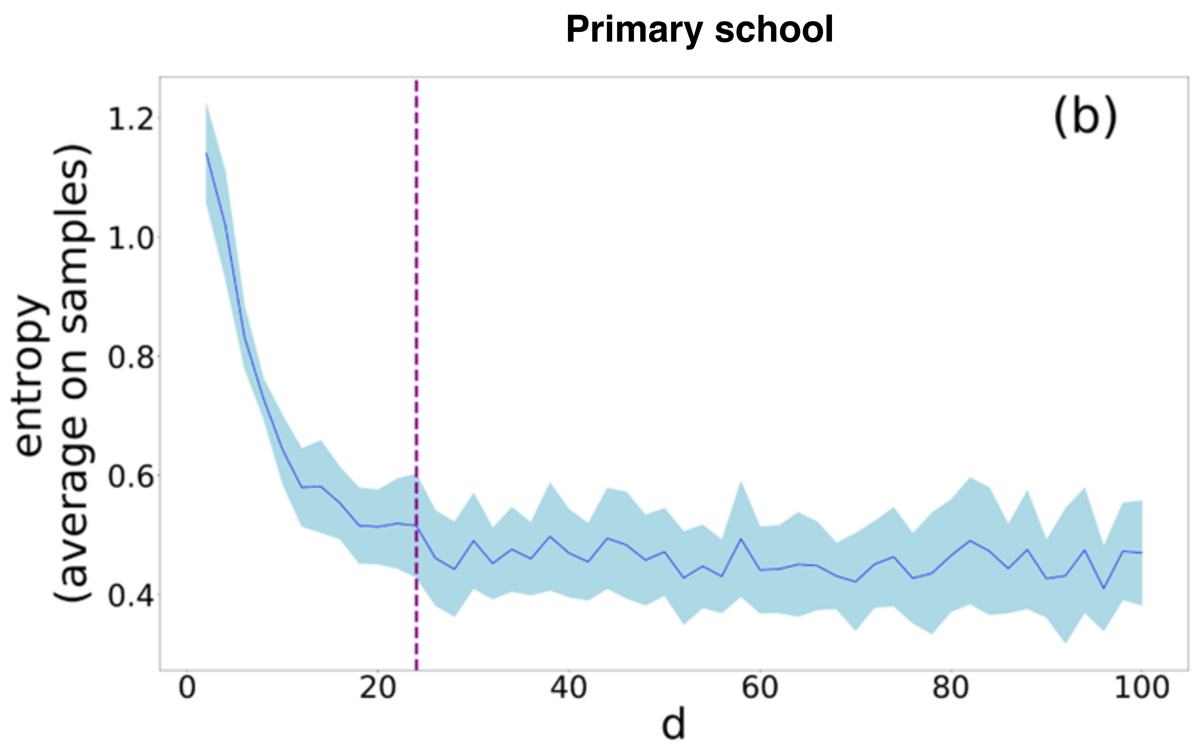
Conference **Primary school**

Weg2Vec - stability

Embedding dimension

- should high enough to capture correlations
- should be low enough to avoid redundancies in the embeddings
- we measure the entropy of euclidean distances between nodes while increasing the dimensions
- once the number of dimensions reaches its optimum nodes will stabilise and entropy becomes constant





Weg2Vec - evaluation

Pearson's correlation coefficients between similarity measures: the **time difference** (in temporal network) and the **euclidean distance** (in embedding) among randomly selected pairs and pairs of adjacent events.

Network	Random corr.	Linked corr.
Conference	0.36±0.01	0.52±0.02
Hospital	0.69±0.02	0.73±0.01
High School	0.40±0.02	0.61±0.01
Primary School	0.34±0.01	0.59±0.01

The method simultaneously captures structural and temporal correlations between events

Weg2Vec - prediction of epidemic size

1. Take a deterministic SI process (β=1)

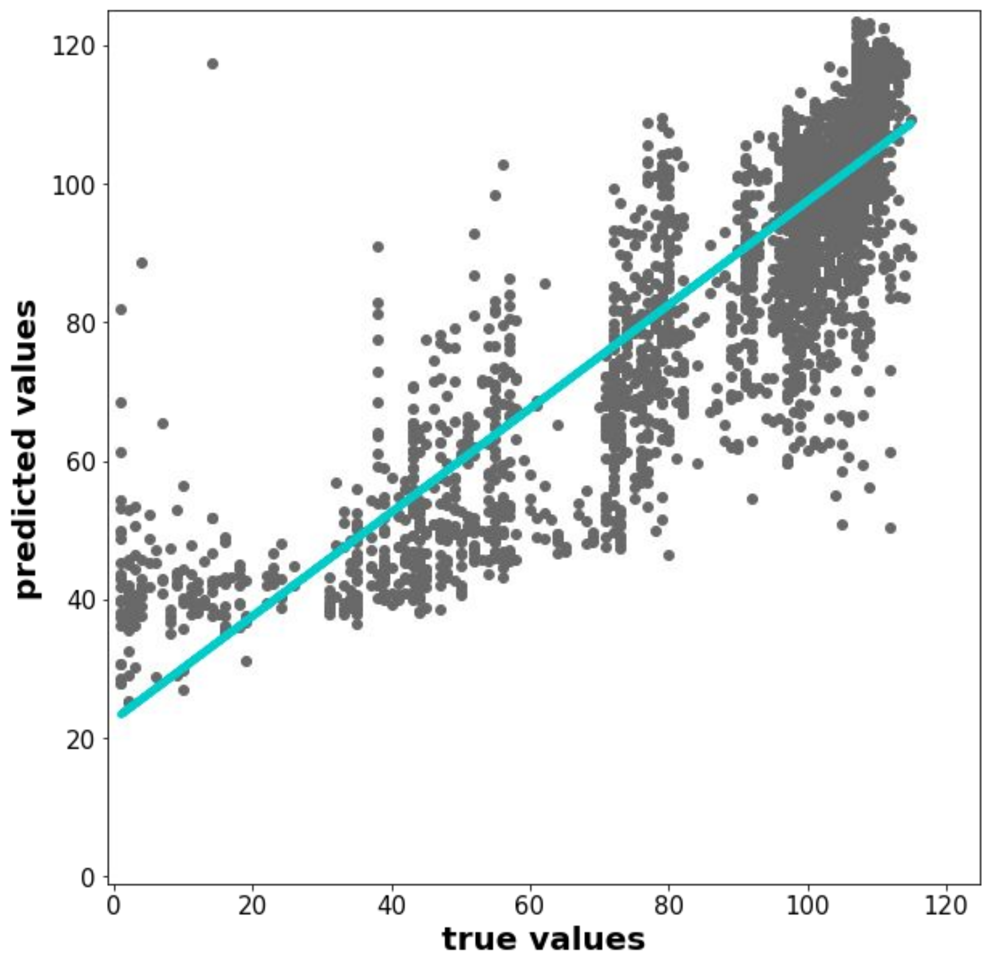
2. Simulate it on the temporal network starting from each event

3. Measure the final epidemic size in each case

4. Take the embedding network ($d=opt$, $s=nb=10$, $\alpha=0.5$)

5. Train a linear regression model on the embedded coordinates and infection sizes of events

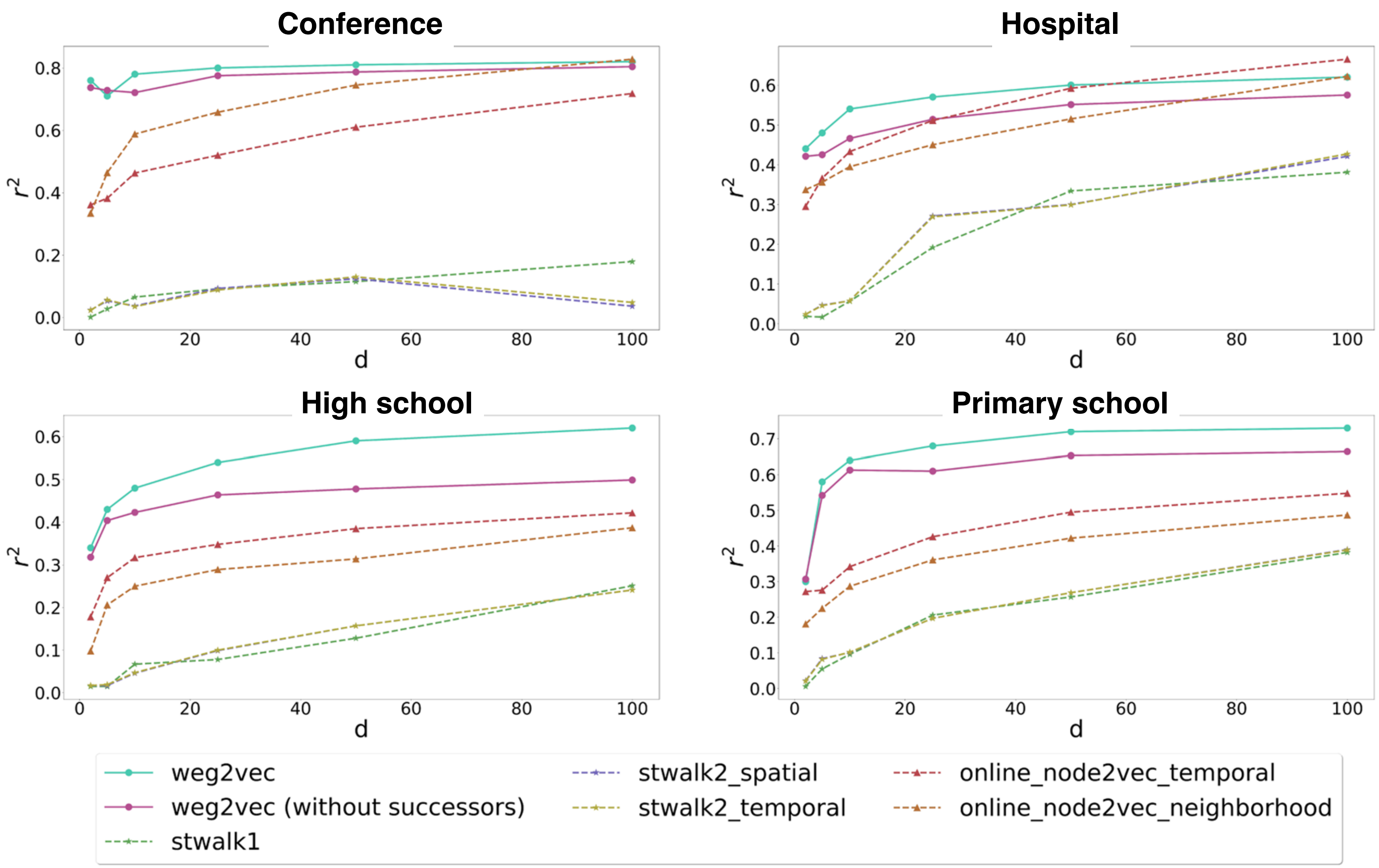
6. Predict the size of epidemic spreading



Data	dimension	r^2
Conference	(d = 20)	0.79 ± 0.01
Hospital	(d = 14)	0.53 ± 0.03
High School	(d = 26)	0.56 ± 0.02
Primary School	(d = 24)	0.68 ± 0.02

Comparison with other methods

- **STWalk[1]** is designed to learn trajectory representations of nodes in temporal graphs by operating with two graph representations: a graph at a given time step and a graph from past time steps. It performs random walks
- **Online-Node2vec[2]** is a node embedding method updating coordinates each time a new event appears in a temporal network. It also applies random walks to generate environments



[1] Pandhre, S., Mittal, H., Gupta, M., & Balasubramanian, V. N. (2018, January). *Proceedings of the ACM India Joint International Conference on Data Science and Management of Data* (pp. 210-219). ACM.

[2] Béres, F., Pálovics, R., Kelen, D., Szabó, D., & Benczúr, A. 7th International Conference on Complex Networks and Their Applications, Cambridge.

Conclusions

Event graphs: static lossless representations of temporal networks

by mapping them as weighted directed acyclic graphs

Weg2Vec - event embedding of temporal networks

- Identification of nodes which influence similar set of other nodes at different times
- Low-dimensional representations based on neighbourhood sampling
- Capture temporal correlations and mesoscale structures

Efficient prediction of spreading outcome

- Outperforming other temporal networks embedding methods

Mapping temporal-network percolation to weighted, static event graphs

Mikko Kivelä, Jordan Cambe, Jari Saramäki & Márton Karsai

Scientific Reports **8**, Article number: 12357 (2018) | Cite this article

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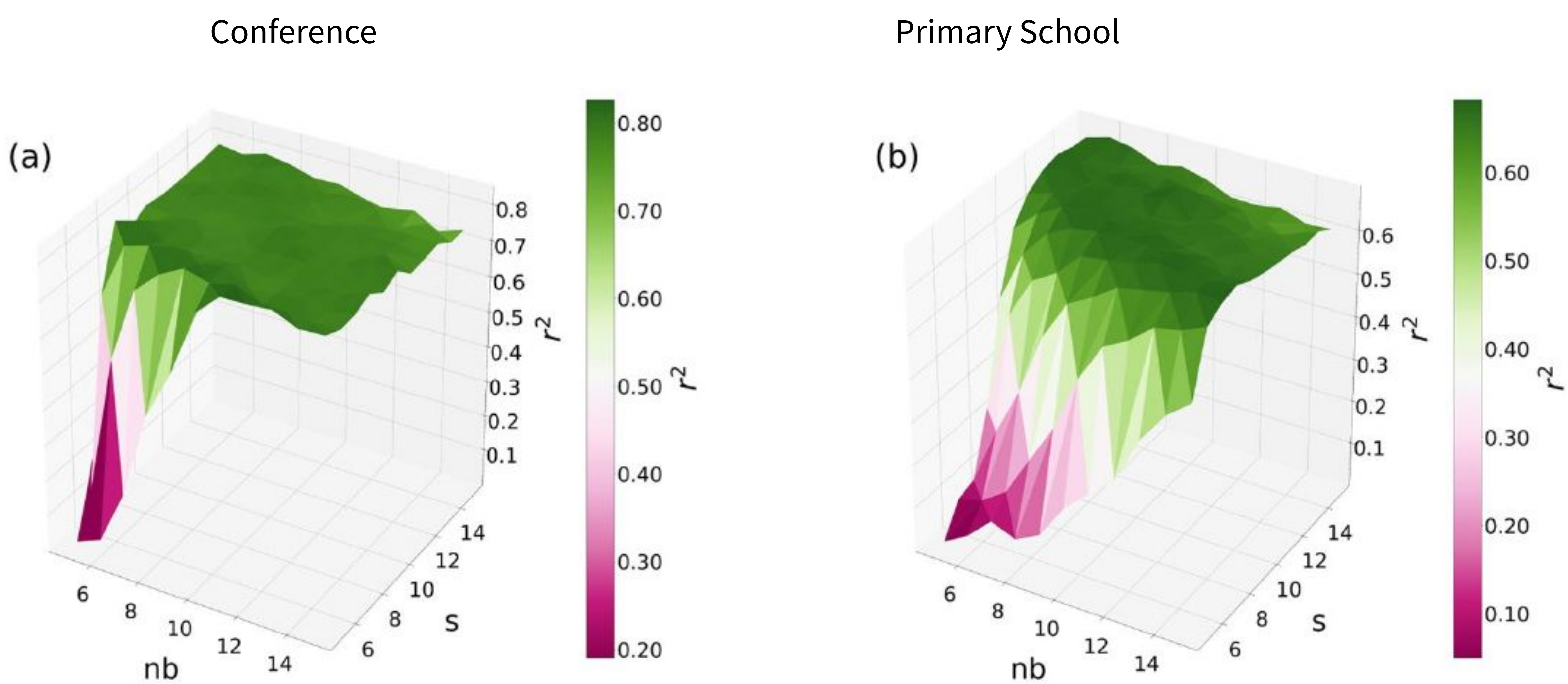
Template slide 1

EFFECT OF THE NEIGHBORHOOD SAMPLING

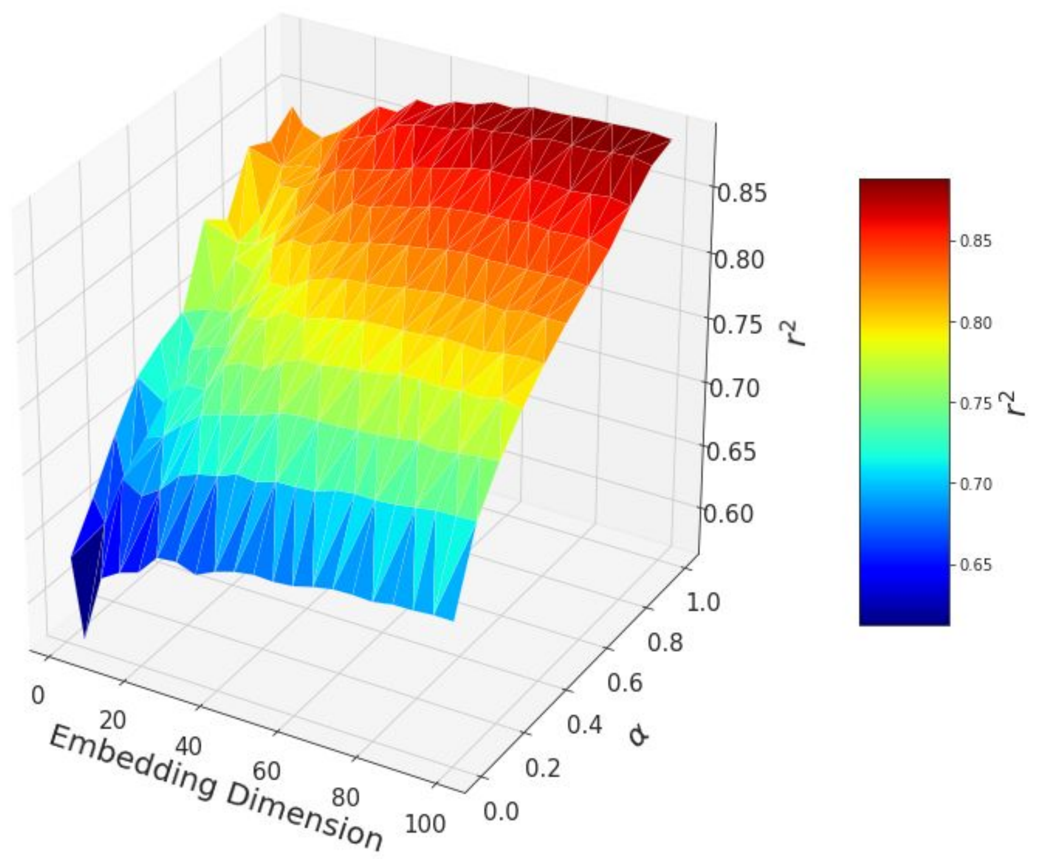


nb= number of environments, s= environment size -> parameters strongly coupled

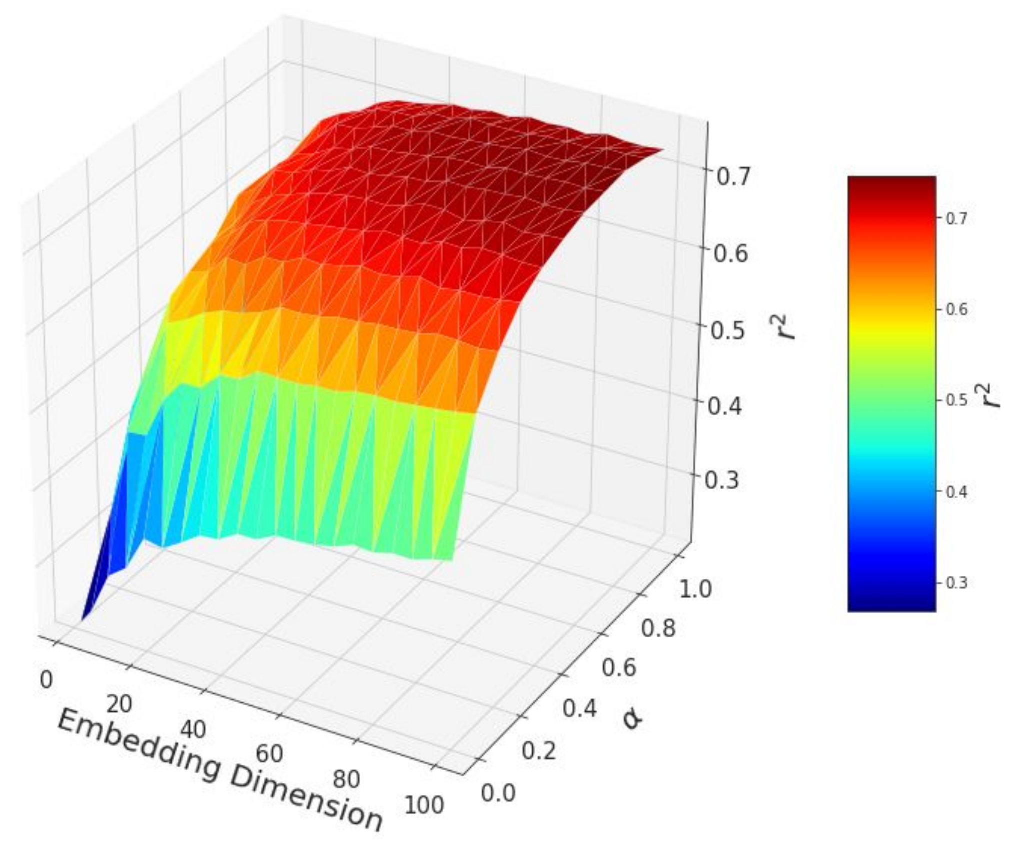
Template slide 1

EFFECT OF THE DIMENSION

Conference



Primary School



α lower values=similarity mainly based on co-occurences