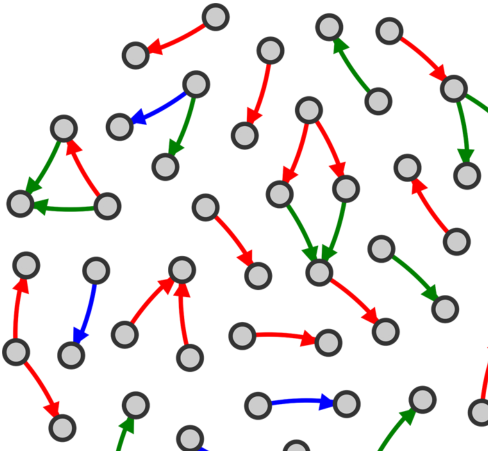
UNIVERSITY OF OXFORD

Mathematical Institute

COLLECTIVE BEHAVIOUR IN TEMPORAL NETWORKS

DOOCN Satellite, CCS18

27th September 2018



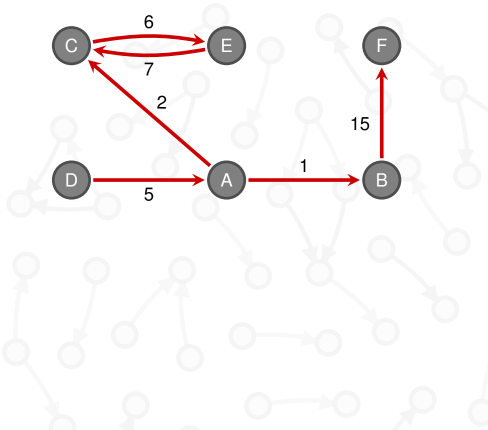
Andrew Mellor

Mathematical Institute

University of Oxford

A sequence of *temporal events*,

$$e_i = (\underbrace{u_i}_{\text{source}}, \underbrace{v_i}_{\text{target}}, \underbrace{t_i}_{\text{time}}, \underbrace{\delta_i}_{\text{duration}})$$



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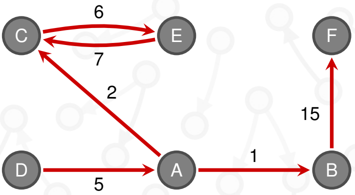

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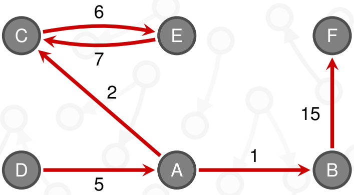

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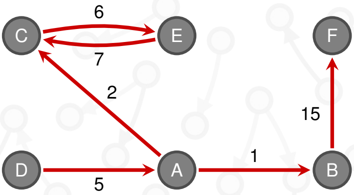

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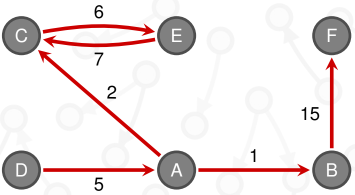

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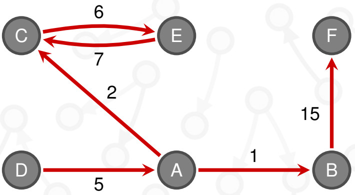
4. Email Correspondence



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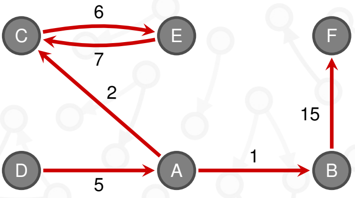

Other Representations:

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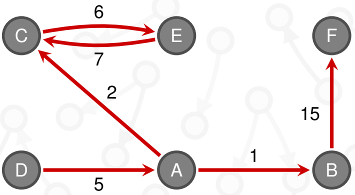
Other Representations:

1. Adjacency matrices

$$(A_k)_{k=1}^{T}$$



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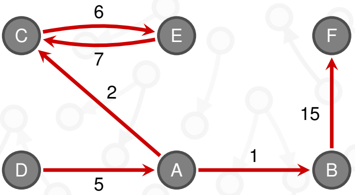

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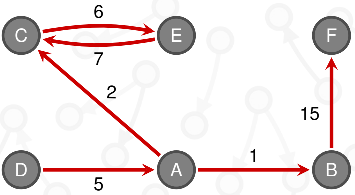
3. Adjacency tensors

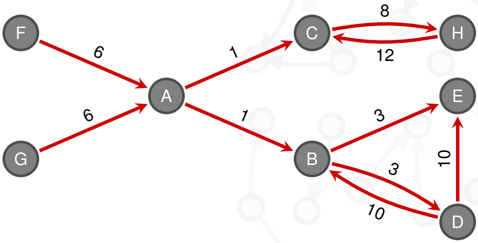


Not all interactions are pairwise, or dyadic.

In these cases we can consider *temporal hyper-events*,

$$e_i = (\underbrace{U_i}_{\text{sources}}, \underbrace{V_i}_{\text{targets}}, \underbrace{t_i}_{\text{time}}, \underbrace{\delta_i}_{\text{duration}})$$

Here a set of sources can interact with a set of targets (undirected hyper-events can also be defined).



EVENT GRAPH

Consider a temporal network $G_T = (V, T, E)$ where $E \subseteq V^2 \times T$ is the set of temporal events.

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Event Graph:

An *event graph* G is a directed static graph given by the tuple $G = (E, f_E)$ where E is a set of temporal events, and $f_E : E \times E \to [0, 1]$ is a binary function which prescribes the edges of the graph.

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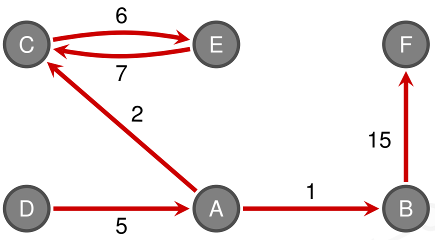
Weighted Event Graph:

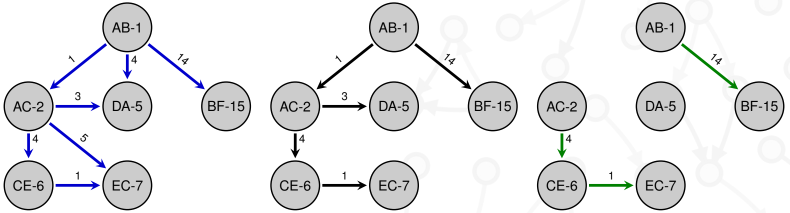
The weighted event graph is topologically equivalent to the event graph only that edges are weighted by the time between the two events.

This amounts to using a weighted joining function

$$f^\tau(e_i, e_j) = \tau_{ij} f(e_i, e_j),$$

where τ_{ij} is the inter-event time.

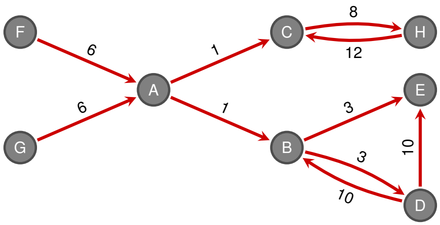




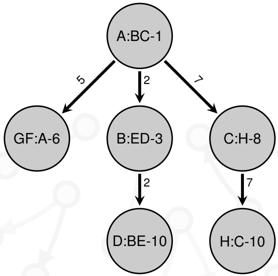
(a) Δt-adjacency

(b) Node-subsequent Δt-adjacency

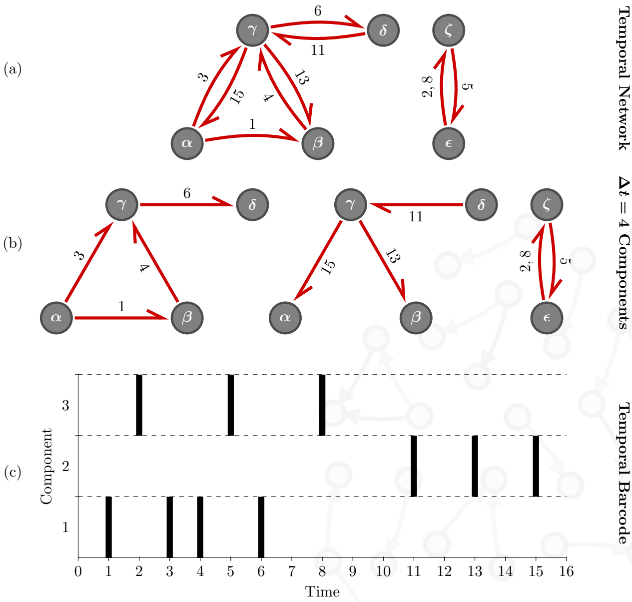
(c) Walk-forming Δt-adjacency



(a) Temporal Hypergraph



(b) Node-subsequent Δt-adjacent event graph

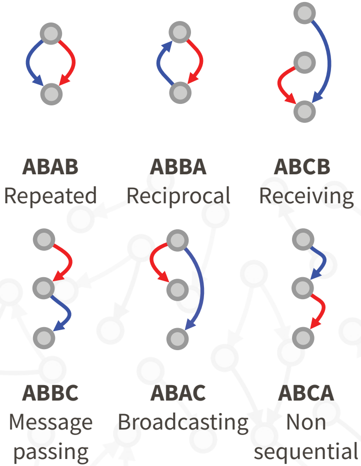


(a)
Temporal Network
6
11
3
15
4
13
1
2, 8
5
(b)
Δt = 4 Components
6
3
4
1
11
15
13
2, 8
5
(c)
Temporal Barcode
Component
Time

Temporal motifs are repeatable patterns observed in the network.

Two adjacent events can have one of 6 node patterns, or motifs.

Each motif is associated with a particular behaviour.



Red events occur before blue events.

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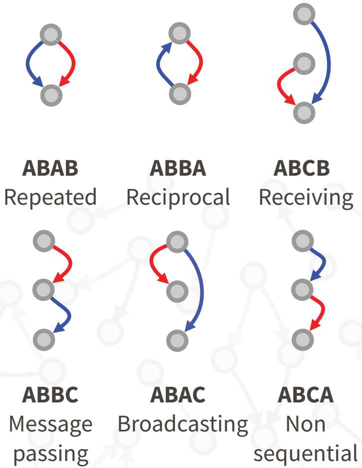

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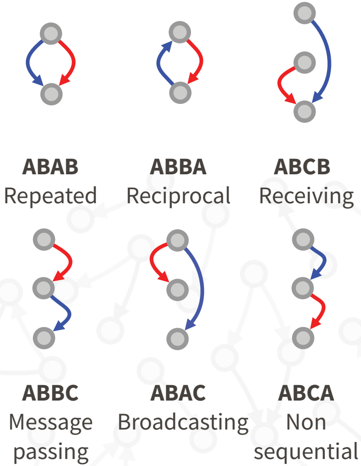
- Higher-order motifs are possible (3, 4 events).



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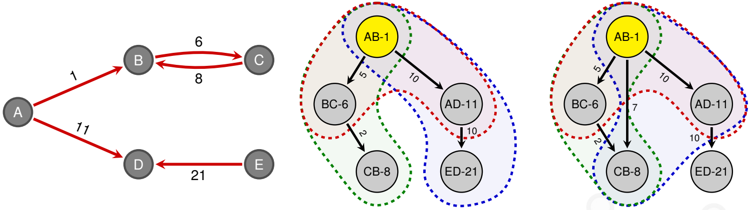
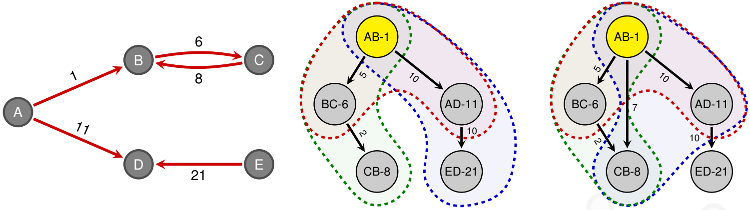


(a) **(b)** Sequential **(c)** Windowed

Figure: Schematics for the calculation of temporal motifs using an event graph.

Sequential	Temporal motifs in time-dependent networks. [*Kovanen et. al.* (2011)]
Windowed	Motifs in temporal networks. [*Paranjape et. al.* (2017)]



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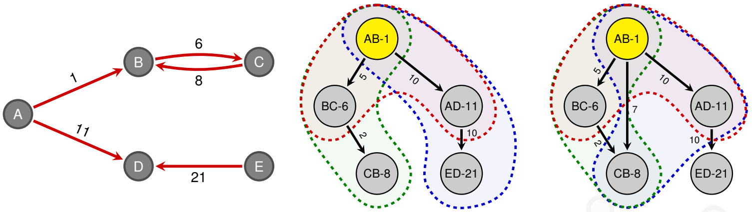
Temporal motifs are closely related to the event graph formalism.



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Temporal motifs are closely related to the event graph formalism.

There are at least two definitions of temporal motifs, both can be expressed as subgraphs of the event graph.

Encode temporal components based on event graph features and aggregate graph features (to capture higher-order effects).

Event Graph Features:

1. Temporal motif distribution
2. Temporal motif entropy
3. Inter-event time entropy
4. Activity

Aggregate Graph Features:

1. Clustering
2. Reciprocity
3. Degree assortativity*

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Normalise feature vectors and perform a hierarchical clustering.

Data collected from Twitter (sampled by keyword using the Twitter Streaming API).

Tweets which contain the word *Emirates* (and common variants).

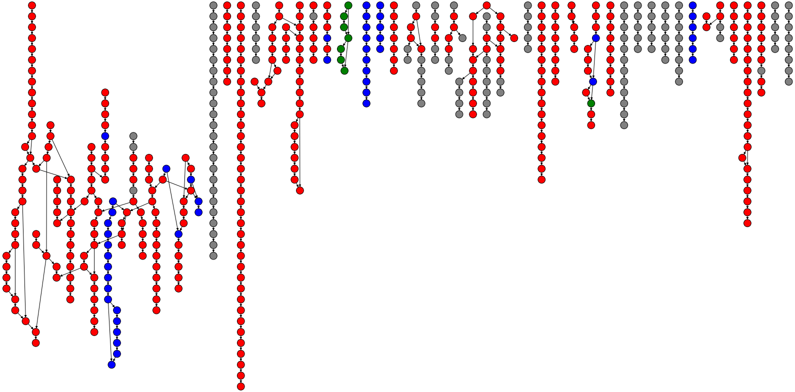
Subsequently collected all the accounts present in the sample and collect all tweets they have participated in during the course of the entire day using the REST API.

We sampled our keyword using the above procedure over one day:

# Tweets	# Events	# Users	% Retweets
161,730	130,360	48,249	52.95

We set the link threshold $\Delta t = 240$s (or four minutes). The 4 minute event graph has 2300 components with at least ten events.

EXAMPLE - EVENT GRAPH



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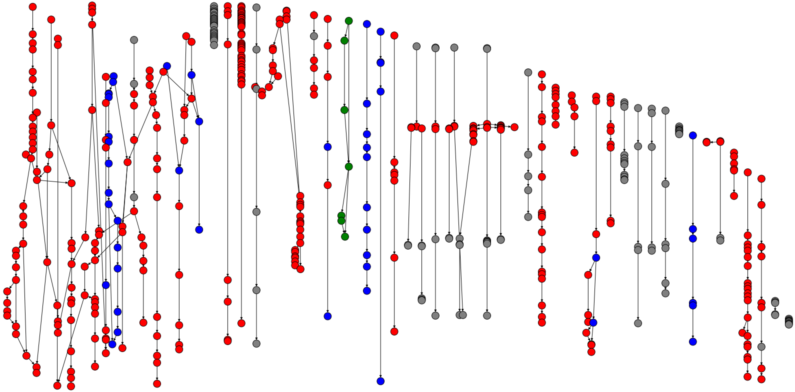

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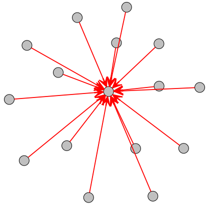
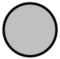
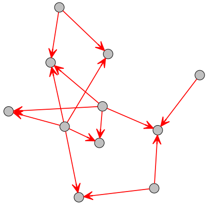


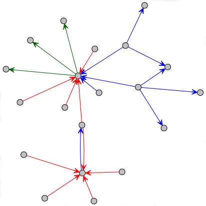
(a) Cluster 1



(b) Cluster 2



(c) Cluster 3



(d) Cluster 4

Aggregate (static) graphs of representative examples from each cluster.

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One way is through correlations of node state timeseries, but can we instead use edge information?

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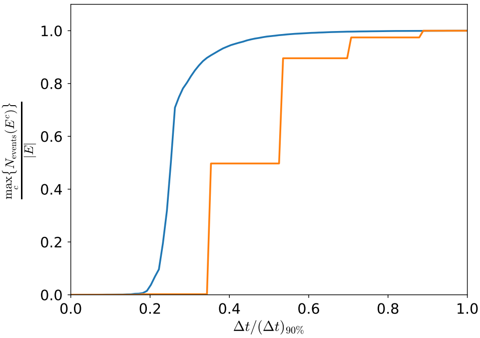
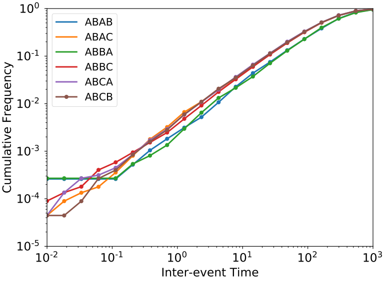
2. An SIS model: Two node states: infected and susceptible. Infected nodes pass on the infection to neighbouring susceptible nodes with rate λ. Infected nodes recover with rate μ.



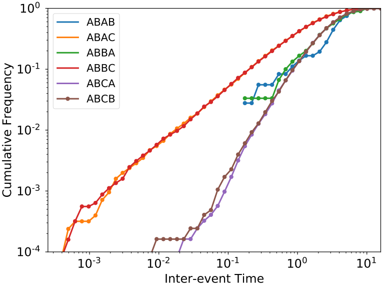
Blue = Random / Orange = SIS

Growth of the largest temporal component as a function of Δt.

One characteristic timescale in the random model, however there are multiple for the SIS model.



(a) Random Process



(b) SIS Process

Inter-event times conditioned on the motif type.

ABBA, ABAB, ABCB and ABCA motifs are far less prominent in the SIS model than the random process.

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2. Use the event graph as a generative model for synthetic temporal networks.

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2. Use the event graph as a generative model for synthetic temporal networks.

3. Compare node-based and event graph based dynamical process inference. Can the two be combined?

Thank you

References:

1. Analysing Collective Behaviour in Temporal Networks Using Event Graphs and Temporal Motifs
   arXiv:1801.10527
2. Event Graphs: Advances and Applications of Second-order Time-unfolded Temporal Network Models
   arXiv:1809.03457

Software: `https://github.com/empiricalstateofmind/eventgraphs`

Website: `andrewmellor.co.uk`

Email: `mellor@maths.ox.ac.uk`

Background

The **inter-event time** between two events e_i, e_j is given by

$$\tau_{ij} = \begin{cases} t_j - t_i & \text{if } t_j > t_i \\ 0 & \text{otherwise.} \end{cases}$$

If events have a duration then

$$\tau_{ij} = \begin{cases} t_j - (t_i + \delta_i) & \text{if } t_j > t_i + \delta_i \\ 0 & \text{otherwise.} \end{cases}$$

IETs are crucial to understanding dynamics and spreading processes on temporal networks.

Two events are Δt-adjacent if they share at least one node, and the time between the two events (inter-event time) is less than or equal to Δt.

As a function, Δt-adjacency can be written as

$$f(e_i, e_j) = \underbrace{(0 < \tau_{ij} \leq \Delta t)}_{\text{Temporal proximity}} \wedge \underbrace{(\{u_i, v_i\} \cap \{u_j, v_j\} \neq \emptyset)}_{\text{Topological proximity}}.$$

In this case, an event can be connected to any number of subsequent events provided the criteria is met.

We can also consider only the subsequent event for each node in the event previous event.

We define the set of all events that node u participates in after time t, given by

$$A_u^t = \{(u', v', t') \in E \text{ s.t. } u \in \{u', v'\} \text{ and } t' > t\}.$$

The joining rule is then given by

$$f_E(e_i, e_j) = (0 < \tau_{ij} \leq \Delta t) \wedge \left(\bigvee_{s \in \{u_i, v_i\}} (j = \min\{k | e_k \in A_s^{t_i}\}) \right).$$

Similarly, we can create a rule for simply the next event that *either* node participates in.

Temporal paths are are an important feature of a temporal network for dynamics.

We can adapt our connection rule to consider only pairs of events where a temporal path is formed (for directed events).

$$f(e_i, e_j) = \underbrace{(0 < \tau_{ij} \leq \Delta t)}_{\text{Temporal proximity}} \quad \wedge \quad \underbrace{(u_j = v_i)}_{\text{Target of event } i = \text{Source of event } j}$$

For certain temporal network processes it might be suitable to introduce a minimum time between events occurring.

For example, on transportation networks (rail, aviation) it is an unrealistic assumption to that a change can be made at a station/airport without first taking time to traverse from one vehicle to another.

$$f(e_i, e_j) = \underbrace{(\Delta t_1 < \tau_{ij} \leq \Delta t_2)}_{\text{Temporal proximity}} \quad \wedge \quad \underbrace{(u_j = v_i) \wedge (v_j \neq u_i)}_{\text{Path forming AND non-backtracking}}$$